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National Bureau of Standards
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Boulder, Colorado 80303

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A GENERALIZED METHOD FOR THE CALIBRATION OF FOUR-TERMINAL-PAIR TYPE DIGITAL IMPEDANCE METERS

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August 1984

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CONTENTS

Page

1. Introduction and Background.... 1
2. Purpose and Scope.... 2
3. Four-Terminal-Pair Interface and Adapters to Other Connector Types.... 3
4. Theoretical Approach.... 5
    4.1 Impedance Relationship through the Adapter.... 5
    4.2 Transformation to Reflection Coefficients.... 7
5. Calibration Experiment.... 8
    5.1 Calibration Curves.... 8
    5.2 Statistical Model for the Calibration Curve.... 10
    5.3 Least-Squares Solution for the Calibration Parameters.... 11
6. Measurement of Unknowns.... 12
    6.1 Measurement of an Unknown Γ.... 12
    6.2 Uncertainty in Γ_{2u}.... 13
    6.3 Transformation to Impedance.... 14
7. Example.... 14
    7.1 Standards Used in Calibration.... 14
    7.2 Least-Squares Estimation of the Calibration Parameters.... 19
    7.3 Measurement of Unknown Γ.... 26
    7.4 Transformation to Z.... 26
    7.5 Calibration Parameters with a 1 m Cable.... 31
8. Summary.... 31
9. References.... 33

Appendix A. FORTRAN Program Listing.... 34

Appendix B. Printout from Program for Data at 1 MHz.... 42

Appendix C. Printout from Program for Data at 10 MHz.... 49

A Generalized Method for the Calibration of Four-Terminal-Pair Type Digital Impedance Meters

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Since the introduction of automated, four-terminal-pair type digital impedance meters, there has been a continuing interest in the development of calibration techniques which would satisfactorily verify the accuracy capabilities of these instruments. Various attempts have been made and all have helped to provide a certain degree of confidence in instrument performance, but until now, a generalized approach with a good mathematical and statistical background has been lacking. This paper describes a calibration procedure having such a background and illustrates its use. The calibration is accomplished through the use of impedance standards which relate instrument readings to the values of the standards through a known functional relationship. The calibration procedure described estimates the parameters associated with the functional relationship and requires the use of a computer. Calibration is accomplished at the reference plane of the impedance standards and any adapter required to connect the standards to the instrument is assumed to be an integral part of the impedance meter.

Key words: calibration; digital impedance meter; impedance; least-squares; measurement; reflection coefficient; uncertainty

1. Introduction and Background

Until about a decade ago, most impedance measurements in the rf range were made by passive methods utilizing either null or resonance principles. Other methods such as the vector impedance meter were available but, in general, where the best accuracy was required, they were not as satisfactory.

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With the introduction of the digital impedance meters utilizing a constant current source in conjunction with an internal resistance standard and a complex voltage ratio detector, came a new era in the measurement of impedance [1]. These new instruments have capabilities which make them far more useful. Their versatility, measurement speed, convenience, accuracy, and relatively low cost have made the older methods obsolete in many situations.

With the rapidly increasing acceptance and use of these instruments, we are encountering the problem of accuracy verification. Early approaches, for the most part, utilized spot checks with available standards and adapters, but at best such methods lack any significant degree of mathematical generality. In other words, even if there is a standard available to provide a check of the instrument accuracy at one point, there is no real assurance that the instrument is correct at some other point where a standard is not available. Thus, there exists a need for improved calibration methods to use with these instruments.

Although there are several different models of the four-terminal-pair (4TP) LCR (inductance, capacitance, resistance) meters in current use, it is believed that the procedure to be described is applicable for each one with the single provision that appropriate standards of impedance are available. The data presented in this report were generated using a Hewlett-Packard[1] Model 4275A LCR meter, serial number 1851J00419. This particular instrument is the property of the Boulder Laboratories of the National Bureau of Standards (NBS).

2. Purpose and Scope

The project is an initial effort to demonstrate the feasibility of the proposed calibration approach. To further develop the procedure to the point where all of the meter characteristics are analyzed and integrated into a complete calibration package is beyond the scope of this initial phase. Within this initial effort we will concentrate on analyzing data taken at frequencies of 1 and 10 MHz. At each of these frequencies two adapter conditions have been studied. The first condition is with a 4TP to 14 mm coaxial adapter connected directly to the front panel of the meter. The second condition utilized the same adapter, but with a 1 m long cable harness inserted between the front panel of the meter and the 4TP to 14 mm coaxial adapter. The latter is a configuration used in remote measurement applications. While there are many other situations and conditions that could have been undertaken, such as temperature, warm-up time, etc., they have been left for future effort if deemed necessary. The purpose here is primarily to learn how to implement the procedure, to study the error sources, and to gain some insight into how it can best be implemented in the future to solve the many and varied calibration needs for impedance meters of this general type.

---

[1] Certain commercial equipment, instruments, or materials are identified in this paper in order to adequately specify the experimental procedure. Such identification does not imply recommendation or endorsement by the National Bureau of Standards, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

3. Four-Terminal-Pair Interface and Adapters to Other Connector Types

In the background discussion, mention was made of the fact that the new generation digital LCR meters employ a measurement principle which is different from the older, more traditional methods. One result of the new method is that the measurement interface (the manner for connecting an unknown impedance for measurement) is a 4TP arrangement of coaxial connectors consisting of a "Hi" and a "Lo" coaxial connector for current and a "Hi" and a "Lo" coaxial connector for voltage. These are arranged in the manner shown in figure 1. Such an arrangement is necessary to facilitate the measurement process, but it does have some important practical disadvantages which include the following:

1. For most measurement applications, it is necessary to provide an adapter to either a single coaxial (a one-port) or to a double coaxial connector (a two-port).[2] Adapters for the measurement of leaded components are also provided by the manufacturer as well as remote probes for in situ measurements of circuit components. Calibration with these in use is not addressed in this effort. To do so would require the development of special standards.

---

[2] A two-port is often called a three-terminal measurement, especially for some capacitance or admittance measurements.

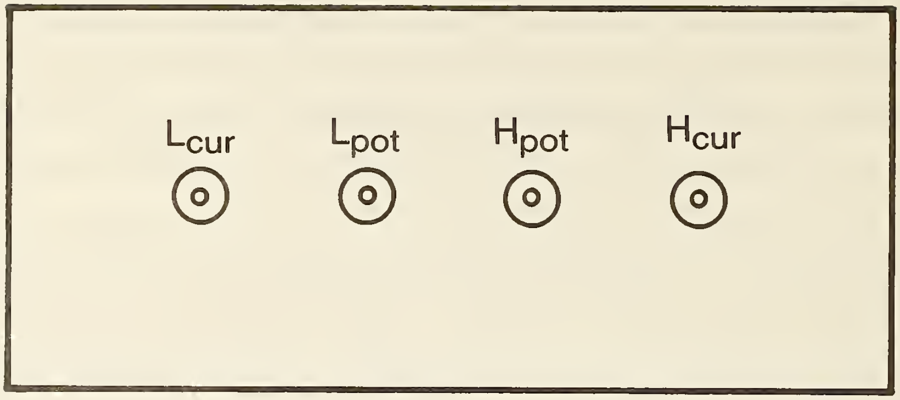

Figure 1. Measurement interface for 4TP-type LCR meter.

2. The calibration problem with these meters presents some difficulties mainly because the measurement interface is a coaxial 4TP configuration for which there are no established standards at NBS. Existing two-terminal (one-port) or three-terminal (two-port) impedance standards cannot be connected directly to the 4TP interface of these meters without some type of an adapter which will itself affect the measurement result.[3] The Hewlett-Packard Company does market standards made specifically for instruments with 4TP-type measurement interfaces. These are resistance and capacitance elements whose values can be measured directly at dc or low frequencies and the high frequency values are then derived from a lumped circuit model by calculation using best estimates for residual impedance contained in the

[3] Because many LCR meters of the 4TP variety have a feature which allows for calibration at any measurement interface utilizing a procedure employing both short- and open-circuit references, the reader may take exception to the statements in 2, above. However, a simple experiment utilizing the 1 m remote measurement feature of the instrument will serve to illustrate the problem. This simple experiment is performed by precalibrating with an adapter connected directly to the instrument and then measuring an unknown device. Following this the 1 m remote measuring harness is inserted between the meter and the adapter and the calibration routine is again executed. When again measuring the same unknown device as before a significantly different result may be observed. This will illustrate the importance of generating correction factors for each measurement configuration.

model. To date, no concentrated attempt has been made at NBS to either duplicate or confirm this approach except for 4TP capacitors [2]. To do so hardly seems appropriate because even with a good absolute calibration at the 4TP interface, there is still a question of the calibration accuracy at the reference plane of an added adapter. It is to be noted, however, that over large portions of the frequency and impedance ranges of these meters, well-designed adapters do not seriously degrade the measurement accuracy.

In this effort, two specific measurement configurations were studied, each employing an adapter from the 4TP interface at the instrument panel to the GR900, 14-mm-type precision coaxial connector. In one configuration the 4TP to 14 mm adapter was attached directly to the instrument and in the second configuration a 1 m remote measurement harness was inserted between the front panel of the meter and the 4TP to 14 mm adapter. In these two configurations the "cable length" switch on the meter panel was set to "0" and "1m", respectively.

4. Theoretical Approach

4.1 Impedance Relationship through the Adapter

The measurement of impedance for devices with 14 mm coaxial connectors requires the use of a 4TP to 14 mm coaxial adapter. The adapter becomes part of the device under test (DUT) and the LCR meter measures the impedance of this DUT in combination with the adapter. What is required is the impedance of the DUT. This is illustrated in figure 2.

In figure 2 the meter measures the impedance at reference plane 1 while the impedance at reference plane 2 is desired. If we view the adapter as a general network with terminal variables v and i, then we can express the relationship between these terminal variables at the two reference planes by the following equations [3].

$$\begin{aligned} v_1 &= Av_2 - Bi_2 \\ i_1 &= Cv_2 - Di_2 . \end{aligned} \tag{4-1}$$

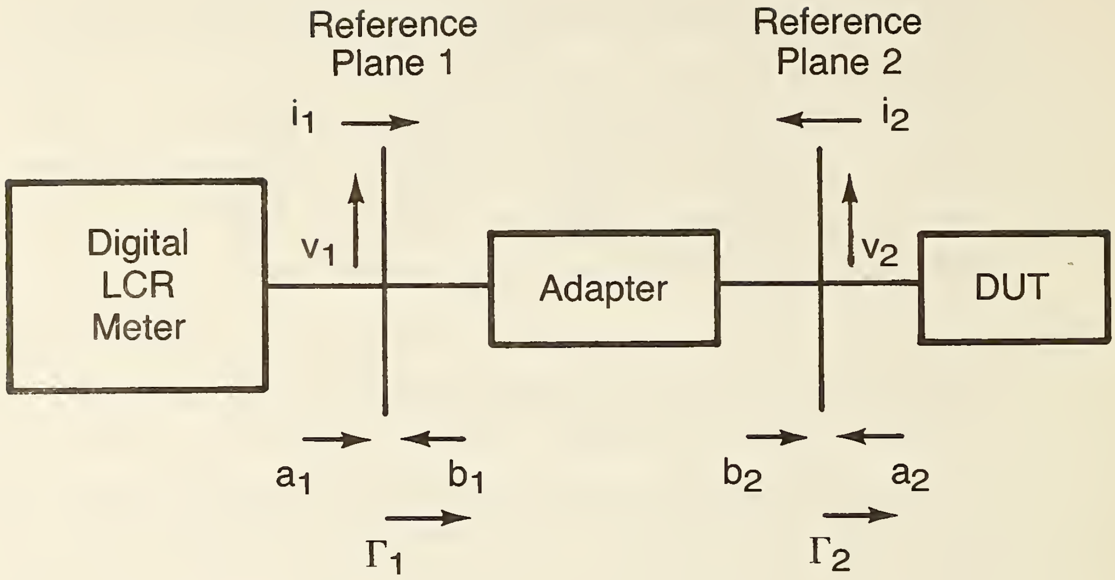

Figure 2. Measurement configuration and symbolism for meter calibration procedure.

Taking the ratio of v_1 to i_1, and dividing the numerator and denominator of the right side by $-Di_2$ gives the following:

$$\frac{v_1}{i_1} = \frac{-\frac{A}{D}\frac{v_2}{i_2} + \frac{B}{D}}{-\frac{C}{D}\frac{v_2}{i_2} + 1}. \tag{4-2}$$

Noting that the ratio v/i is equal to the impedance Z, and letting A' = -A/D, B' = B/D, C' = -C/D we get:

$$Z_1 = \frac{A'Z_2 + B'}{C'Z_2 + 1}. \tag{4-3}$$

In theory, if there were no measurement error, we could obtain the unknown parameters A', B', C' by measuring the impedance Z_1 for three known values of Z_2 and solving three complex equations like eq (4-3) for the three complex unknowns.

However, in practice, Z_1 is measured with error. To account for this error the following statistical model is used:

$$Z_{1i} = \frac{A'Z_{2i} + B'}{C'Z_{2i} + 1} + e_i, \tag{4-4}$$

where

Z_{1i} is the measured impedance for the ith standard,

Z_{2i} is the known impedance of the ith standard,

e_i is a complex random error in the ith measured impedance, and

A', B', C' are the unknown parameters.

One approach to estimate the parameters in eq (4-4) is via the method of least squares. However, one assumption for the valid use of ordinary least squares is that the e_i have a constant variance. Because the relationship in eq (4-4) is assumed valid for all values of Z in the complex plane, the constant variance assumption means the effect of the random errors is independent of Z. The application of least squares to eq (4-4) resulted in large residuals for large values of impedance. This suggests that the variance of the e_i is not constant for all values of Z but is a function of Z. A common procedure used to accommodate nonconstant variance of the error term e_i is to transform the data so that the assumption of equal variance is valid. In many instances this transformation has to be obtained empirically. Fortunately, there is a transformation based on physical relationships that achieves the required result.

4.2 Transformation to Reflection Coefficients

If we choose a, b as terminal variables (see fig. 2) instead of v, i, we are led to the following network equations [3]

$$b_1 = r_{11}a_2 + r_{12}b_2$$

$$a_1 = r_{21}a_2 + r_{22}b_2. \tag{4-5}$$

Taking the ratio of b_1 to a_1 and dividing the numerator and denominator of the right side by $r_{22}b_2$ gives:

$$\frac{b_1}{a_1} = \frac{\left(\frac{r_{11}}{r_{22}}\right) \frac{a_2}{b_2} + \frac{r_{12}}{r_{22}}}{\left(\frac{r_{21}}{r_{22}}\right) \frac{a_2}{b_2} + 1}. \tag{4-6}$$

If we let $\alpha = r_{11}/r_{22}$, $\beta = r_{12}/r_{22}$, $\gamma = r_{21}/r_{22}$, and note that the ratio b_1/a_1 is the reflection coefficient Γ_1 at reference plane 1 in figure 2 and a_2/b_2 is the reflection coefficient Γ_2 at reference plane 2, then we can rewrite eq (4-6) as follows

$$\Gamma_1 = \frac{\alpha \, \Gamma_2 + \beta}{\gamma \, \Gamma_2 + 1}. \tag{4-7}$$

The model in eq (4-7) transforms the reflection coefficient Γ_2 to a reflection coefficient Γ_1 through the parameters α, β, and γ. Thus, the model is a linear fractional transformation of Γ_2 to Γ_1 and the estimation of the parameters α, β, γ can be thought of as "calibrating" the adapter. The statistical methods to estimate these unknown parameters are discussed in the next section.

5. Calibration Experiment

5.1 Calibration Curves

Calibration is a process of intercomparing an unknown with a standard and assigning a value to the unknown based on the value of the standard. If the calibration is desired over an extended regime of interest, and a functional relationship can be shown to exist between the standard and unknown, then a calibration curve can be used to assign values to the unknown based on the values of the standard. For example, the functional relationship between y and x might be

$$y = \alpha + \beta x, \tag{5-1}$$

where y is a reading or measurement from an instrument, and x is the known value of the standard.

The calibration of unknowns is affected by a two-step process. First, the calibration experiment produces data on n standards, say x_i, y_i; i = 1, . . .n where x_i is the known value of the ith standard and y_i is the corresponding measurement made by the instrument on the ith standard. In the example, these data are used to estimate the parameters α and β in eq (5-1). After obtaining the estimates for the parameters, denoted by $\hat{\alpha}$ and $\hat{\beta}$, and measuring a future unknown, y_f, the corresponding estimated value for x is found by solving eq (5-1). This gives

$$x_f = \frac{y_f - \hat{\alpha}}{\hat{\beta}}, \tag{5-2}$$

where x_f is the estimated value. This procedure is illustrated in figure 3. The dots are the coordinates of the x,y data obtained in the calibration experiment. The solid line is the estimated calibration curve. A future reading, y_f, is related to the standards by drawing a line horizontally from this value and "reading" the x coordinate of the point of intersection. This gives the estimated value x_f.

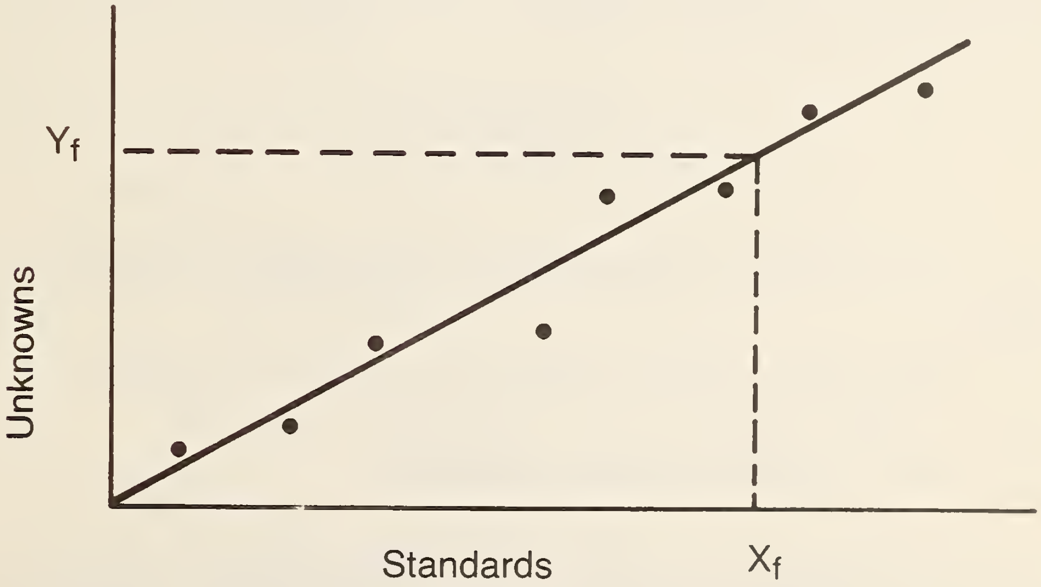

Figure 3. Schematic diagram of a calibration curve.

A final requirement is an assessment of the uncertainty in x_f. There are several statistical issues concerning the construction of interval estimates for x_f, but these are beyond the scope of this paper. One approach is to use propagation of error formulas to estimate the variance of x_f in eq (5-2). This procedure is used to access the uncertainty for the calibration of LCR impedance meters and is discussed in a later section.

5.2 Statistical Model for the Calibration Curve

The model in eq (4-7) which relates the two reflection coefficients is analogous to the simple linear model presented in the last section. Geometrically, it is more difficult to visualize the relationship in eq (4-7) because the variables Γ_1 and Γ_2 as well as the parameters α, β, γ are complex quantities. However, this does not pose significant problems analytically.

In section 4.2 we defined Γ_2 to be the reflection coefficient at reference plane 2 in figure 2, while Γ_1 is the reflection coefficient at reference plane 1. If we choose a device for which we know the reflection coefficient Γ_2 and connect it to the adapter we can measure Γ_1, which is the reflection coefficient obtained by the LCR meter. We assume that the relationship in eq (4-7) holds for all values of Γ inside the unit circle, however due to measurement error in Γ_1 the relationship will not be exact. This leads to the following statistical model:

$$\Gamma_{1i} = \frac{\alpha\, \Gamma_{2i} + \beta}{\gamma\, \Gamma_{2i} + 1} + e_i, \tag{5-3}$$

where

Γ_{1i} is the LCR meter reading when the ith standard is connected to the adapter,
Γ_{2i} is the known reflection coefficient for the ith standard,
e_i is a complex random error of measurement in Γ_{1i}, and
σ, β, and γ are unknown complex parameters.

In the remainder of this paper it is assumed that the errors e_i are random in nature and that any errors in the standard values are negligible compared to the e_i. This assumption will be discussed in connection with the evaluation of the procedures to be presented in a later section.

5.3 Least-Squares Solution for the Calibration Parameters

The least-squares solution for the parameters in eq (5-3) is one that minimizes the following quantity:

$$S = \sum_{i=1}^{n} \left| e_i \right|^2, \tag{5-4}$$

where

$$e_i = \Gamma_{1i} - \left(\frac{\alpha\,\Gamma_{2i} + \beta}{\gamma\,\Gamma_{2i} + 1}\right), \tag{5-5}$$

and n is the number of standards.

Since the e_i are complex numbers, the least-squares solution is implemented in the following manner. S in eq (5-4) can be rewritten as:

$$S = \sum_{i=1}^{n} \left[\mathrm{Re}(e_i)^2 + \mathrm{Im}(e_i)^2\right], \tag{5-6}$$

where we denote the real and imaginary parts of a complex number by Re and Im, respectively. Substitution of eq (5-5) into eq (5-6) gives

$$S = \sum_{i=1}^{n} \left\{\left[\mathrm{Re}(\Gamma_{1i}) - \mathrm{Re}\left(\frac{\alpha\,\Gamma_{2i} + \beta}{\gamma\,\Gamma_{2i} + 1}\right)\right]^2 + \left[\mathrm{Im}(\Gamma_{1i}) - \mathrm{Im}\left(\frac{\alpha\,\Gamma_{2i} + \beta}{\gamma\,\Gamma_{2i} + 1}\right)\right]^2\right\}. \tag{5-7}$$

If we now make the following notational changes:

$y_i = \mathrm{Re}(\Gamma_{1i})$ $\qquad i = 1, 3, 5, \cdots, 2n - 1$

$f_i(x_i, \underline{\theta}) = \mathrm{Re}\left(\frac{\alpha\,\Gamma_{2i} + \beta}{\gamma\,\Gamma_{2i} + 1}\right)$ $\qquad i = 1, 3, 5, \cdots, 2n - 1$

$\varepsilon_i = \mathrm{Re}(e_i)$ $\qquad i = 1, 3, 5, \cdots, 2n - 1$

$y_i = \mathrm{Im}(\Gamma_{1i})$ $\qquad i = 2, 4, 6 \cdots 2n,$

$f_i(x_i, \underline{\theta}) = \mathrm{Im}\left(\frac{\alpha\,\Gamma_{2i} + \beta}{\gamma\,\Gamma_{2i} + 1}\right)$ $\qquad i = 2, 4, 6 \cdots 2n,$

$\varepsilon_i = \mathrm{Im}\ (e_i)$ $\qquad i = 2, 4, 6 \cdots 2n,$

and

$$\underline{\theta} = [\mathrm{Re}(\alpha), \mathrm{Im}(\alpha), \mathrm{Re}(\beta), \mathrm{Im}(\beta), \mathrm{Re}(\gamma), \mathrm{Im}(\gamma)].$$

We can rewrite the model in eq (5-3) as

$$y_i = f_i(x_i, \underline{\theta}) + \varepsilon_i. \tag{5-8}$$

The least-squares solution for $\underline{\theta}$ in eq (5-8) is one that minimizes

$$S = \sum_{i=1}^{2n} (y_i - f_i(x_i, \underline{\theta}))^2 \tag{5-9}$$

and it can be shown that the expressions in eqs (5-7) and (5-9) are equivalent. It should be noted that the expressions $f_i(x_i, \underline{\theta})$ are nonlinear in the parameters, i.e., α, β, and γ, thus eq (5-9) has to be solved by nonlinear iterative least squares techniques. This poses no great difficulty as software is readily available that can solve nonlinear least-squares problems [4].

The nonlinear least-squares solution has been implemented at NBS and tested using data obtained at 1 and 10 MHz. The results are discussed in section 7.

6. Measurement of Unknowns

6.1 Measurement of an Unknown Γ

After obtaining estimates for the calibration parameters α, β, and γ, it is possible to estimate the reflection coefficient Γ_2 for an unknown DUT. Letting Γ_{1u} be the LCR reading for an unknown DUT, then the estimated reflection coefficient, Γ_2, is obtained by inverting eq (4-7) for Γ_2 which gives:

$$\Gamma_{2u} = \frac{\Gamma_{1u} - \hat{\beta}}{\hat{\alpha} - \Gamma_{1u}\hat{\gamma}} \tag{6-1}$$

where

Γ_{1u} is the measured Γ when the DUT is connected to the adapter,

$\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ are the estimates of the calibration parameters, and

Γ_{2u} is the estimated Γ for the DUT.

6.2 Uncertainty in Γ_{2u}

In order to estimate the uncertainty in Γ_{2u} we need to know the uncertainty in the parameter estimates $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ and in the meter reading Γ_{1u}. These are the quantities which appear on the right side of eq (6-1). The least-squares solution described in section 5.3 provides an estimate of the standard deviation of Γ_1. This is given by

$$\hat{\sigma} = S/[2(n - p)], \tag{6-2}$$

where

S = sum of squares in eq (5-7) at the solution,

n = number of standards in the calibration experiment, and

p = number of parameters estimated.

The estimated standard deviations for $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ are also obtained from the least-squares solution. We will not describe how we obtain these estimates, but will denote these by S_α, S_β, and S_γ.[6]

An approximation for the standard deviation of Γ_{2u} can be obtained by using propagation of error formulas [5]. Application of this procedure to eq (6-1) gives this standard deviation, denote this by $S(\Gamma_{2u})$. The details of these procedures are beyond the scope of this report but will be demonstrated with sample data in the next section.

---

[6] The quantities Γ, α, β, λ are complex numbers. We can treat these analytically as two real variables. Thus, each complex quantity actually has two standard deviations, one associated with the real part and one associated with the imaginary part. We use the notation $S\alpha$, for example, to represent the standard deviation for either the real or imaginary part of α for notational convenience.

6.3 Transformation to Impedance

It was mentioned in section 4.1 that the objective was to obtain the impedance for the DUT. The calibration procedure described so far gives the reflection coefficient for the DUT and its associated standard deviation. These are Γ_{2u} and $S(\Gamma_{2u})$. The impedance is obtained from the reflection coefficient by the transformation

$$Z_{2u} = \frac{50(1 + \Gamma_{2u})}{(1 - \Gamma_{2u})}. \tag{6-3}$$

Finally, we require an estimate of the uncertainty in Z_{2u}. We can obtain an estimate of the standard deviation of Z_{2u} by the application of propagation of error formulas. We have the necessary quantities to accomplish this, namely Γ_{2u} and $S(\Gamma_{2u})$. We will now present the application of the procedures described to sample data obtained at 1 and 10 MHz.

7. Example

7.1 Standards Used in Calibration

The calibration experiment discussed in section 5.2 required known values for n standards. At 1 MHz the standards used to calibrate the meter were a short-circuit, open-circuit, 50 and 100 Ω terminations, a 1000 pF capacitor, and inductors of 1, 2.5, 5, 10, and 25 μH. These values relative to 50 Ω are displayed in figure 4. The standards used at 10 MHz were a short-circuit, an open-circuit, 50 and 100 Ω terminations, a 1 μH inductor, and capacitors of 200 and 1000 pF. These values relative to 50 Ω are displayed in figure 5.

At each frequency the standard was connected to the adapter and the impedance was measured[4] by the LCR meter. These were then transformed to reflection coefficients by:

$$\Gamma = \frac{Z - 50}{Z + 50}. \tag{7-1}$$

These data are presented in tables 1 and 2.

---

[4] Prior to the measurements the manufacturer's suggested calibration procedure was not used. The meter was turned on and allowed to warm up as specified in the operating manual. For these tests, the LCR meter was not "zeroed" on open and short circuit terminations. This procedure was used because it was felt that the stored set of default values provided a more stable reference for the measurements than the daily measurements of opens and shorts.

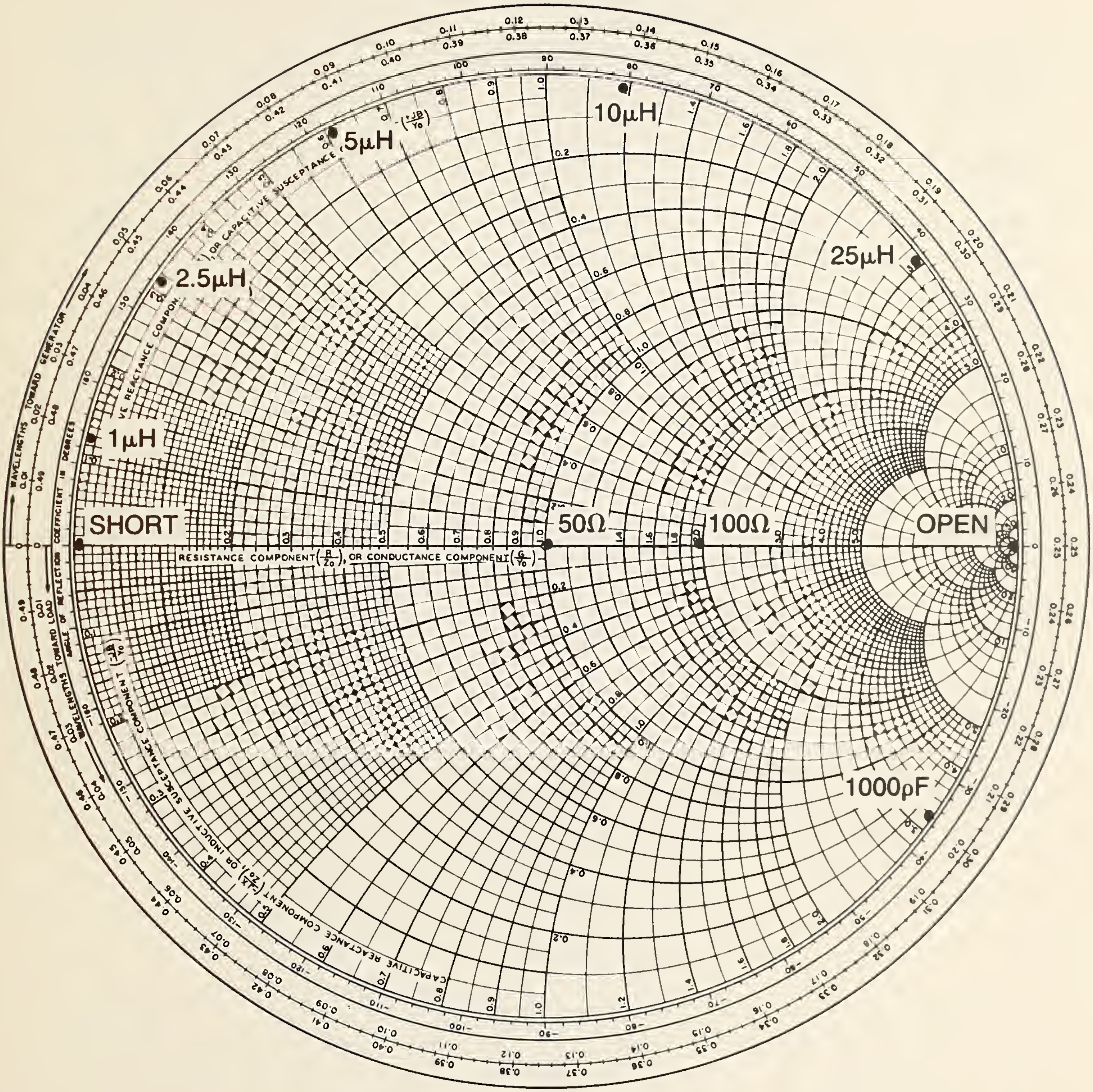

Figure 4. The reflection coefficients of the various impedance standards used at 1 MHz.

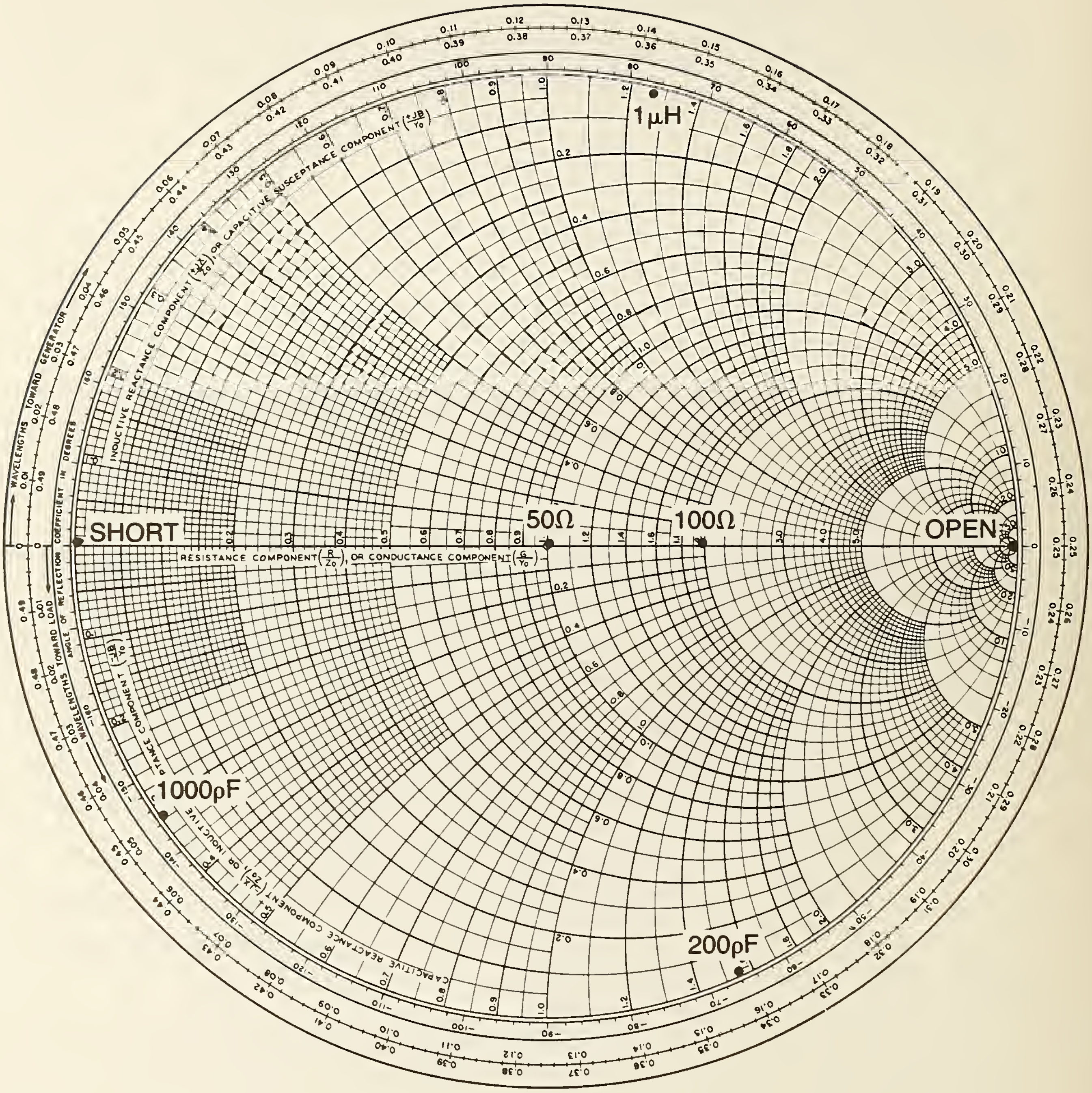

Figure 5. The reflection coefficients of the various impedance standards used at 10 MHz.

Table 1. Calibration data, 1 MHz; with adapter connected to LCR meter.

Standard		Impedance (Ω)		Reflection coefficient	
		Standard value Z_2	LCR reading Z_1	Standard value Γ_2	LCR reading Γ_1
Short	Re	0.00000	0.00646	-1.00000	-0.99973
	Im	0.00000	0.11945	0.00000	0.00478
50 Ω	Re	50.02500	50.06500	0.00025	0.00065
	Im	0.08730	0.05400	0.00087	0.00054
100 Ω	Re	99.83000	99.93900	0.33258	0.33306
	Im	-0.19790	-0.16900	-0.00088	-0.00075
Open	Re	0.00000	-25.72000	1.00000	1.00000
	Im	-159000.00000	-73680.00000	-0.00063	-0.00136
1000 pF	Re	0.00000	0.00000	0.82016	0.81944
	Im	-159.06700	-158.72000	-0.57214	-0.57316
1 μH	Re	0.07970	0.09770	-0.96789	-0.96620
	Im	6.07200	6.17770	0.23860	0.24246
2.5 μH	Re	0.16980	0.20430	-0.81709	-0.81431
	Im	15.62000	15.70530	0.56574	0.56757
5 μH	Re	0.26180	0.28000	-0.44740	-0.44829
	Im	30.76200	30.71800	0.88586	0.88482
10 μH	Re	0.47440	0.50000	0.17351	0.17365
	Im	59.66100	59.67400	0.97691	0.97646
25 μH	Re	1.41370	1.39000	0.79400	0.79421
	Im	149.38000	149.43600	0.59853	0.59841

Table 2. Calibration data, 10 MHz; with adapter connected to LCR meter.

Standard		Impedance (Ω)		Reflection coefficient	
		Standard value Z_2	LCR reading Z_1	Standard value Γ_2	LCR reading Γ_1
Short	R_e	0.00000	0.03240	-1.00000	-0.99767
	I_m	0.00000	1.13870	0.00000	0.04547
50 Ω	R_e	50.06400	49.87800	0.00064	-0.00114
	I_m	0.00294	0.90600	0.00003	0.00908
100 Ω	R_e	99.93000	99.52100	0.33307	0.33122
	I_m	-1.25200	-0.87550	-0.00557	-0.00392
Open	R_e	0.00000	-21.96000	0.99998	0.99995
	I_m	-15915.00000	-72730.00000	-0.00628	-0.01375
1000 pF	R_e	0.00000	0.05569	-0.82755	-0.84936
	I_m	-15.35900	-14.18000	-0.56139	-0.52390
1 μH	R_e	0.03070	0.28747	0.22774	0.23130
	I_m	63.05100	63.34900	0.97323	0.96835
200 pF	R_e	0.00000	0.06723	0.43364	0.40692
	I_m	-79.55100	-77.04000	-0.90108	-0.91259

Table 3. Least-squares estimates of calibration parameters; with adapter connected to LCR meter.

	1 MHz		10 MHz	
	Real	Imaginary	Real	Imaginary
α	0.99983	-0.00218	0.99823	-0.02415
S_α	0.00040	0.00040	0.00127	0.00127
β	-0.00065	0.00066	-0.00511	0.00852
S_β	0.00036	0.00036	0.00110	0.00110
γ	-0.00120	-0.00111	-0.00716	-0.00973
S_γ	0.00041	0.00041	0.00130	0.00130
Residual standard deviation $\hat{\sigma}$	0.00096		0.00285	
Degrees of freedom	14		8	

7.2 Least-Squares Estimation of the Calibration Parameters

A Fortran program, written on a CDC CYBER 750, solves the least-squares formulation presented in section 5.3. This program was then run using the data in tables 1 and 2 to test the procedure. The program listing is presented in appendix A, and program printouts for data at 1 and 10 MHz are given in appendices B and C, respectively. The estimated parameters and standard deviations using these data are presented in table 3.

Table 3 gives the real and imaginary parts of the estimated parameters and their associated standard deviations. For example, at 1 MHz the real part of α was estimated to be 0.99983 with a standard deviation of 0.0004. The residual standard deviation, $\hat{\sigma}$, is computed from eq (6-2).

If the model in eq (5-3) is adequate then the residuals are estimates of the random errors, e_i, given in eq (5-5). The residuals r_i are computed by

$$\mathrm{Re}(r_i) = \mathrm{Re}(\Gamma_{1i}) - \mathrm{Re}\left(\frac{\hat{\alpha}\,\Gamma_{2i} + \hat{\beta}}{\hat{\gamma}\,\Gamma_{2i} + 1}\right) \qquad (7\text{-}1)$$

$$\mathrm{Im}(r_i) = \mathrm{Im}(\Gamma_{1i}) - \mathrm{Im}\left(\frac{\hat{\alpha}\,\Gamma_{2i} + \hat{\beta}}{\hat{\gamma}\,\Gamma_{2i} + 1}\right). \qquad (7\text{-}2)$$

These residuals are displayed in figures 6 through 9. In these figures the residuals computed in eqs (7-1) and (7-2) are plotted on the z axis against the standard values which are on the x-y plane. These plots illustrate the magnitude of the residuals as a function of location of the standard values for Γ in the unit circle. Other graphs of these residuals are presented in figures 10 and 11. In these graphs the imaginary part of the residual is plotted on the y axis against the real part of the residual on the x axis. The residuals appear as vectors in these graphs and should be uniformly distributed around the origin. An outlier, or abnormally large vector, would need to be investigated before the estimation results could be accepted. It is not within the scope of this paper to discuss all the diagnostic procedures associated with the examination of these residuals. These will be developed further in later work.

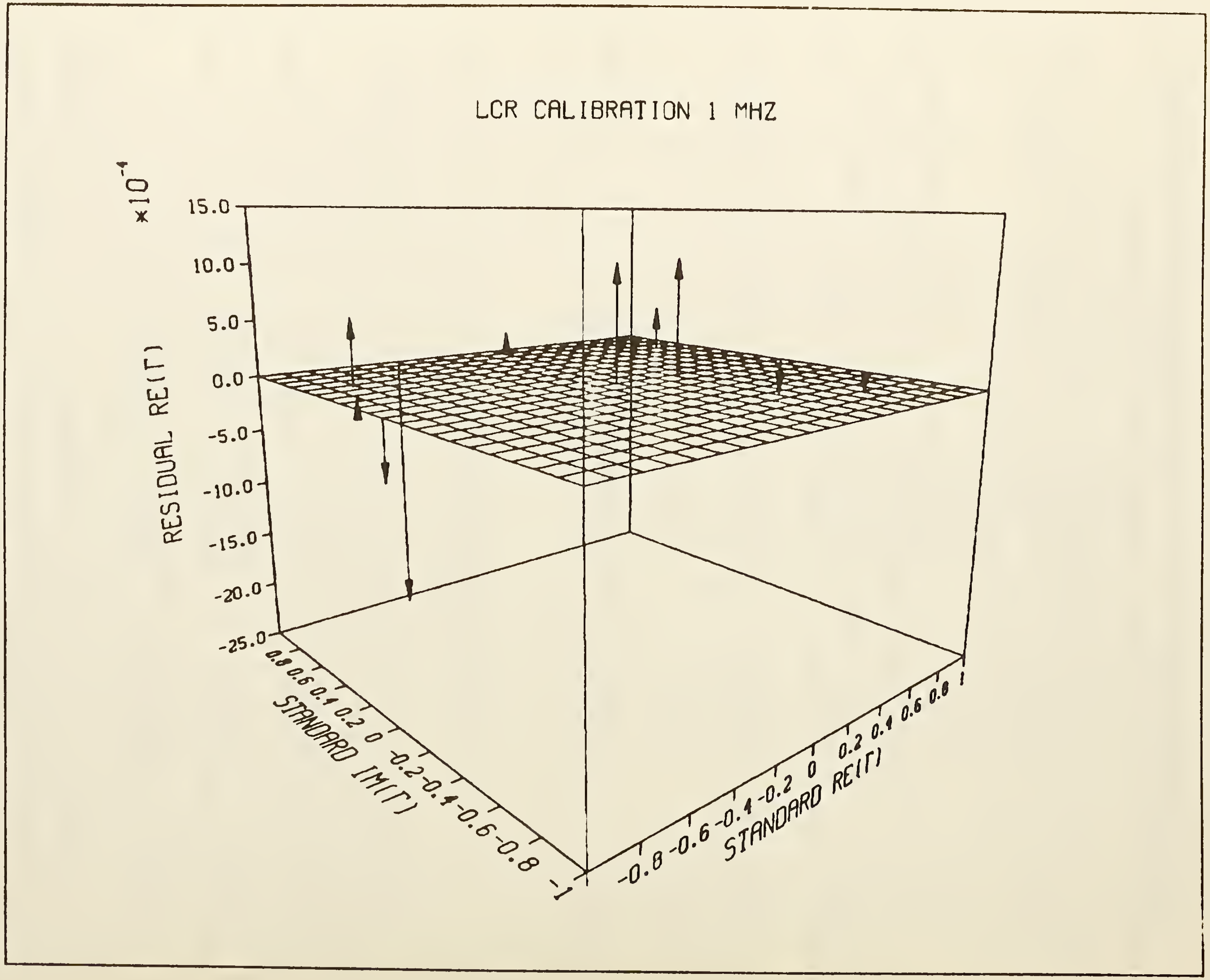

Figure 6. The real part of the residuals at 1 MHz.

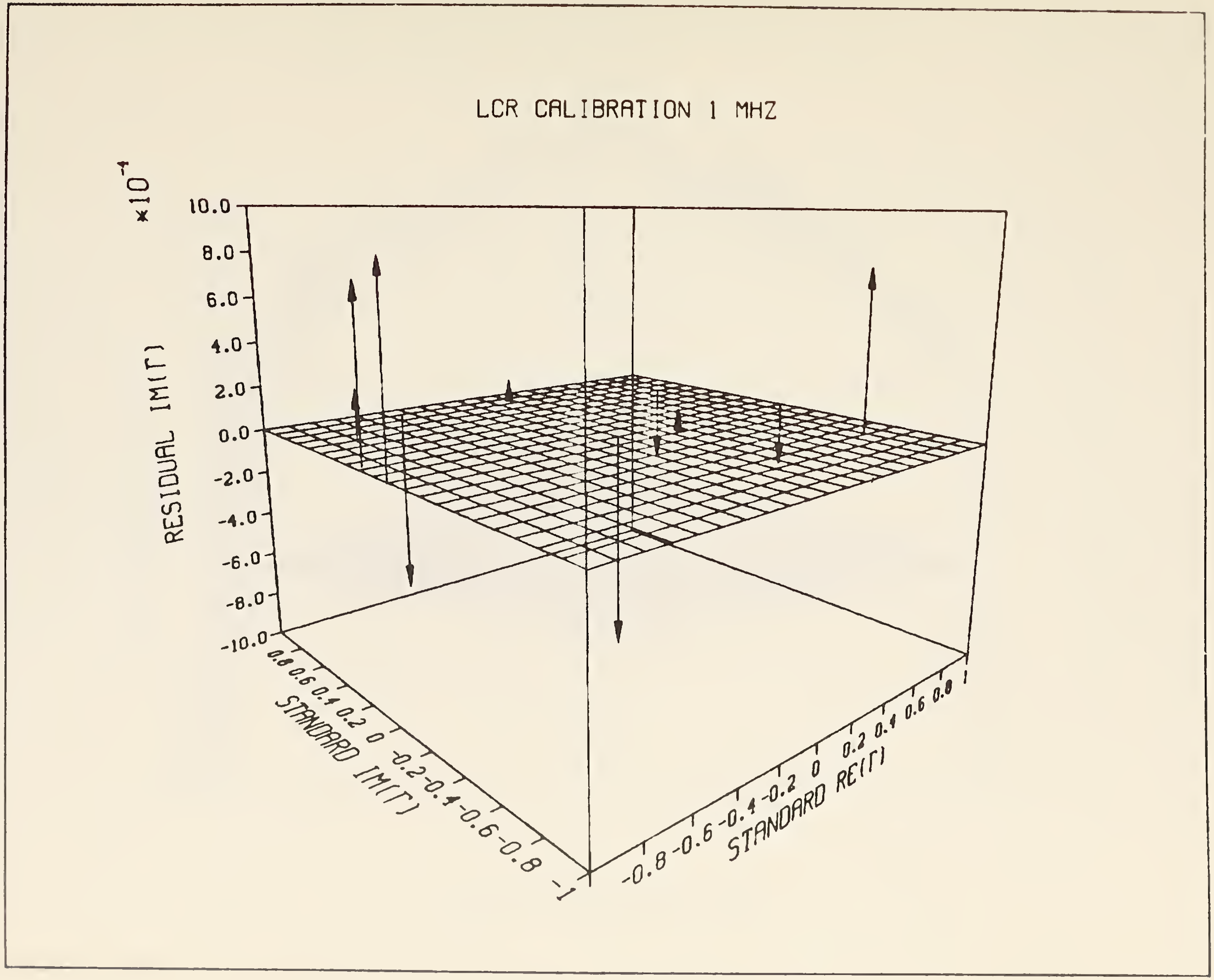

Figure 7. The imaginary part of the residuals at 1 MHz.

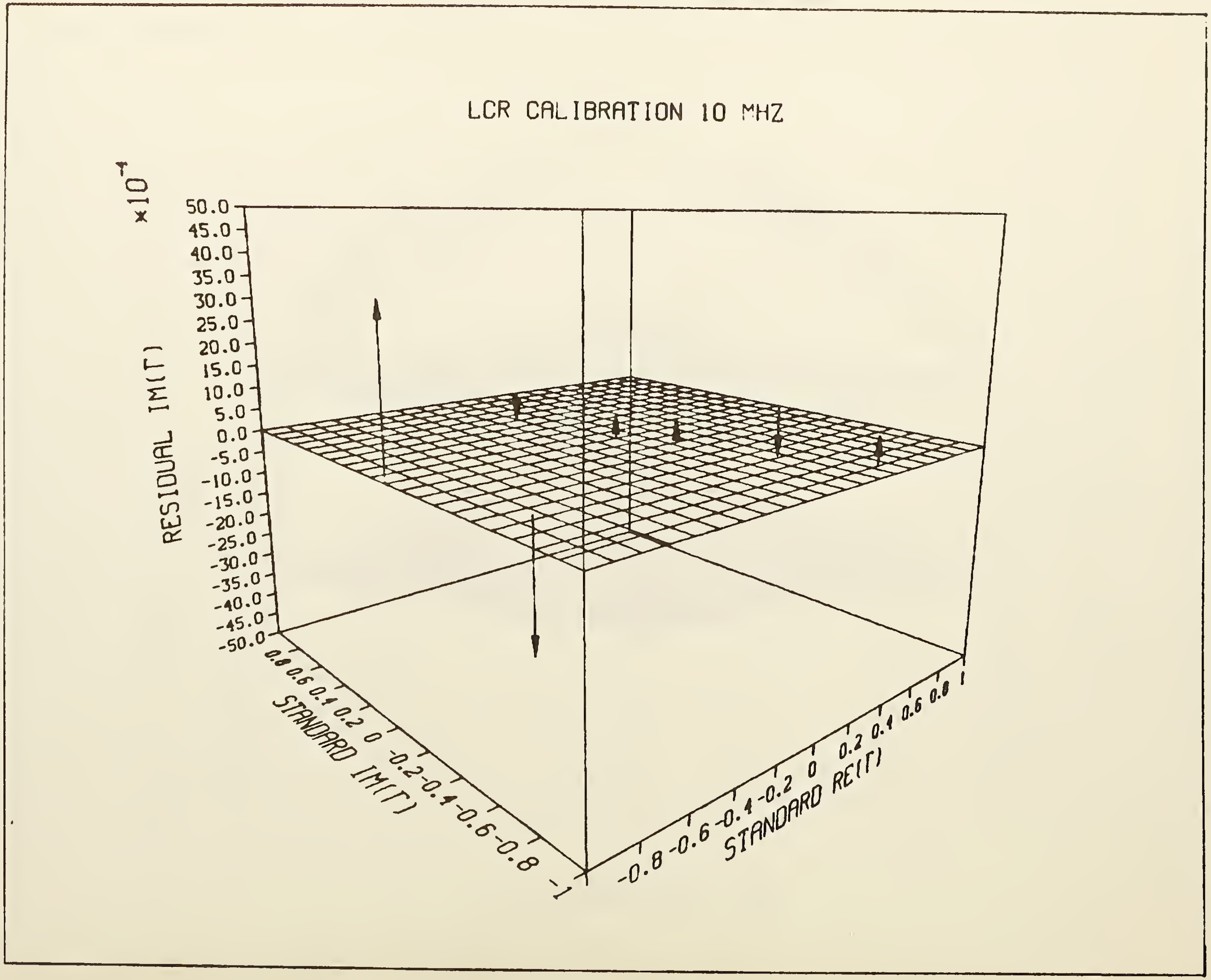

Figure 8. The real part of the residuals at 10 MHz.

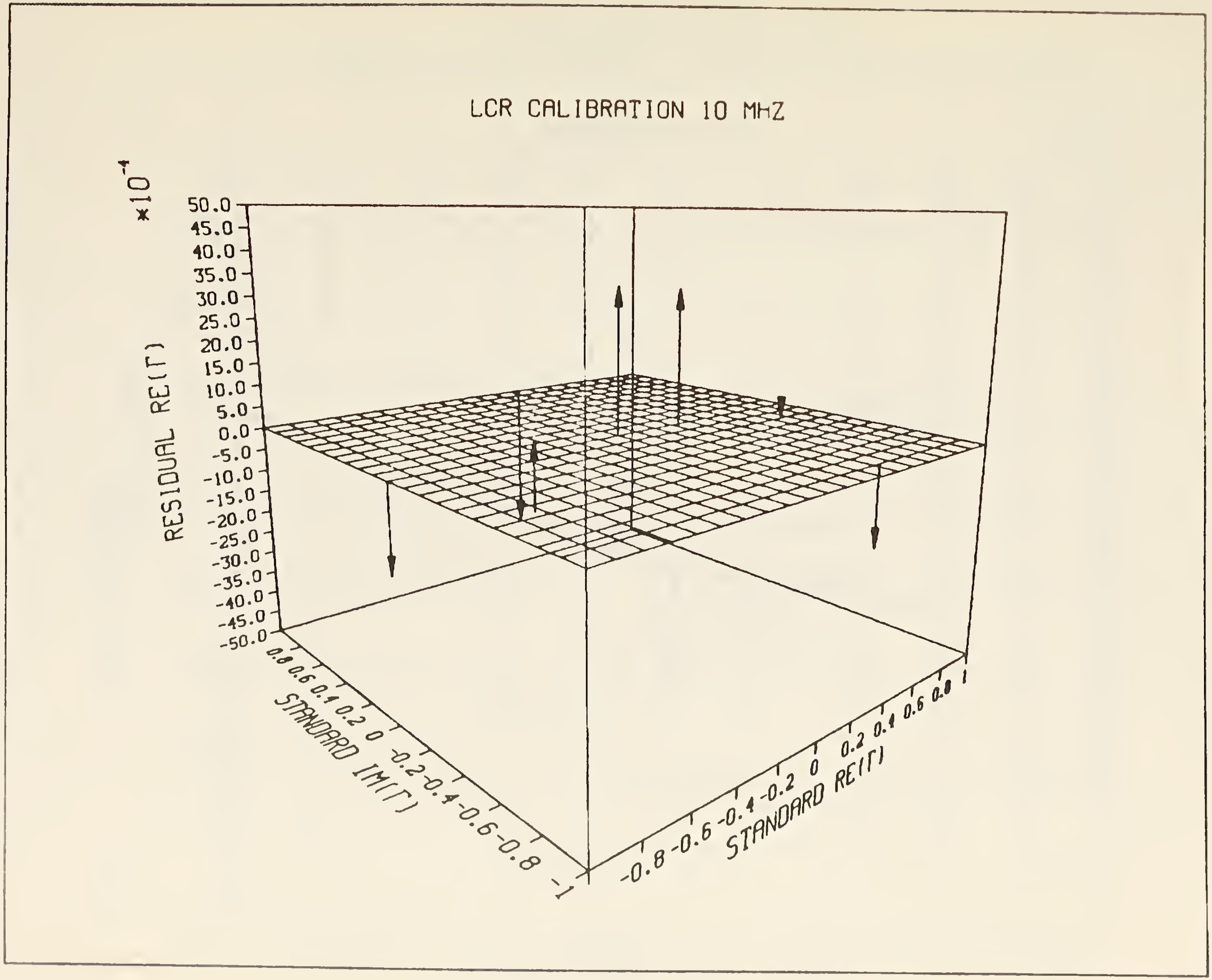

Figure 9. The imaginary part of the residuals at 10 MHz.

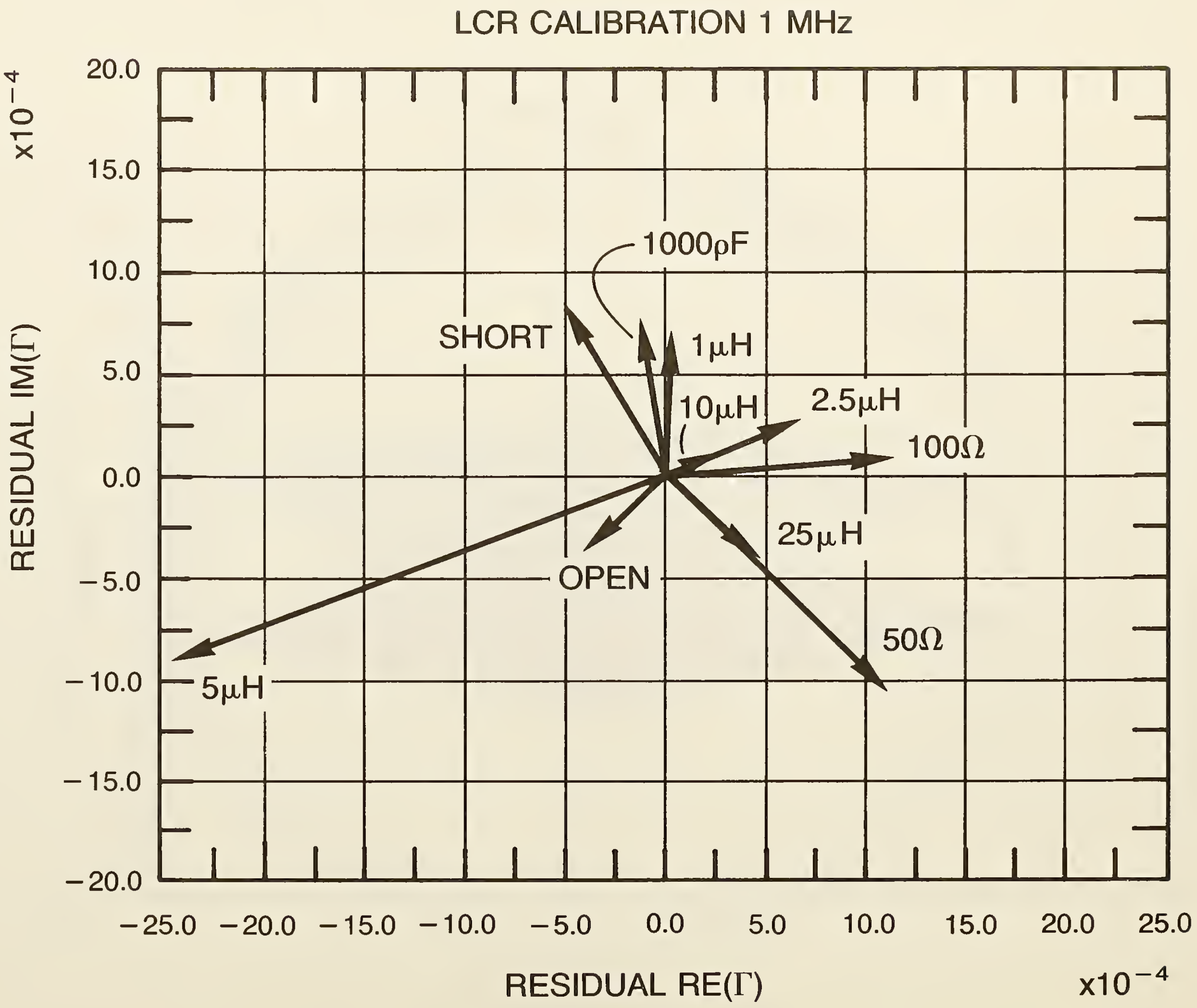

Figure 10. Plot of residuals: Imaginary versus real at 1 MHz.

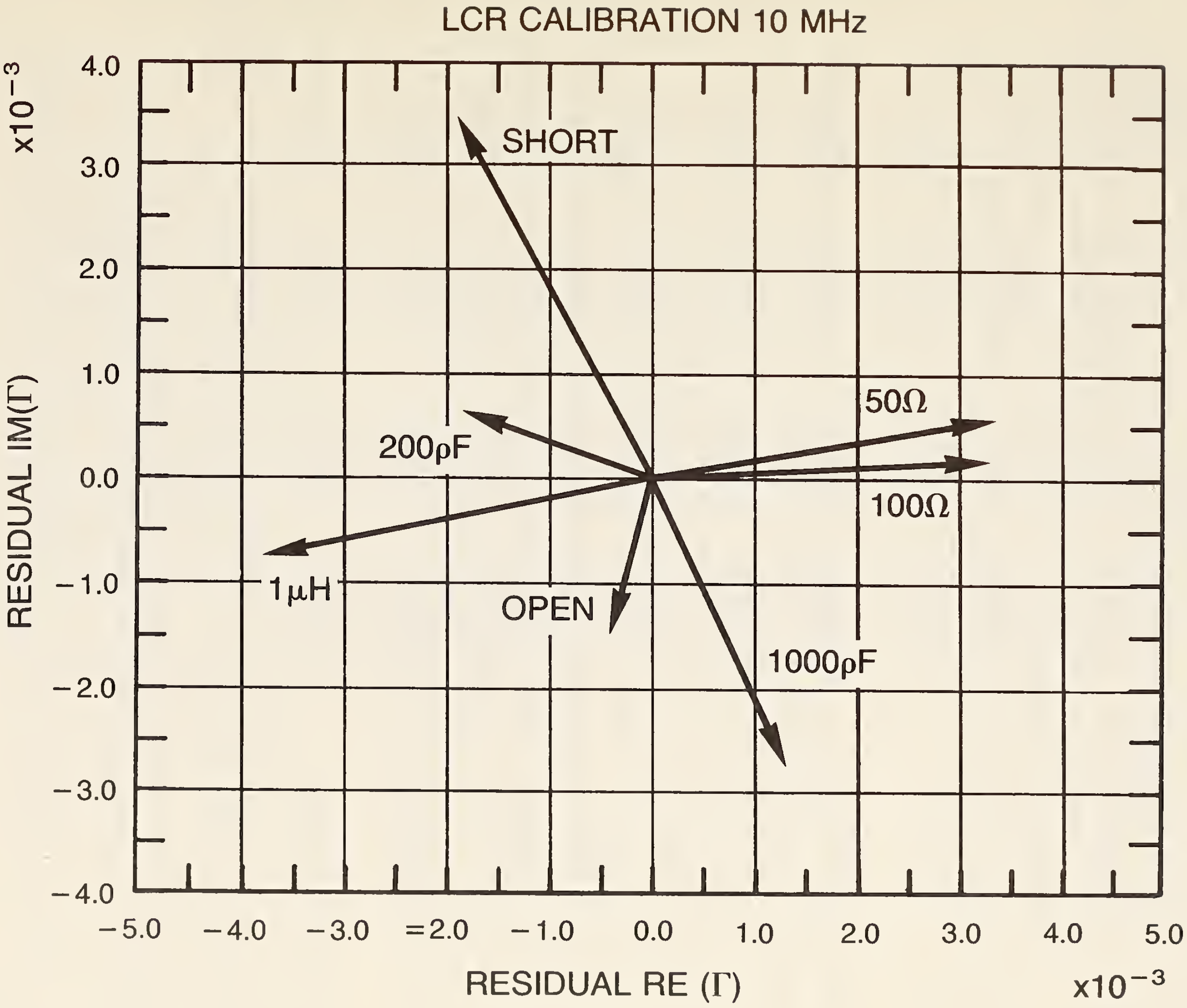

Figure 11. Plot of residuals: Imaginary versus real at 10 MHz.

7.3 Measurement of Unknown Γ

Having obtained estimates of the calibration parameters we can now apply the methods presented in section 6.1 and 6.2 to obtain the reflection coefficient and the associated standard deviation for an unknown device. To demonstrate the procedure a program was written which evaluates eq (6-1) to obtain Γ_{2u} and its standard deviation $S(\Gamma_{2u})$. The data used for Γ_{1u} are the original LCR meter readings which are presented in tables 1 and 2 as Γ_1. The "estimated values," Γ_{2u}, and the known standard values Γ_2 which they estimate are presented in tables 4 and 5. The difference between Γ_{2u} and Γ_2 is also tabulated.

In these tables the real and imaginary part of Γ_{2u} and Γ_2 are presented and the numbers right underneath these are estimates of the errors in these quantities. For Γ_{2u} these are the quantities $S(\Gamma_{2u})$ computed by propagation error formulas. For Γ_2 these are estimates of the systematic errors in the standards. The differences between Γ_{2u} and Γ_2 can be compared to the standard deviation of Γ_{2u}, $S(\Gamma_{2u})$ to get a feel for the adequacy of the proposed procedures. Examination of these differences indicates that the assumption of the error e_i being mostly random is not likely. The e_i contain a systematic component due to the large systematic errors in Γ_2. Therefore, the standard deviations $S(\Gamma_{2u})$ are not really estimates of the random error in Γ_{2u} since a significant systematic error is present.

7.4 Transformation to Z

Having obtained Γ_{2u} we can now transform these to impedances by eq (6-3), and estimate the standard deviation from propagation of error techniques. This was done using the values for Γ_{2u} in tables 4 and 5. The results, Z_{2u} corresponding to Γ_{2u} are presented in tables 6 and 7, along with the known impedances, Z_2. The entries in these tables are in the same format as those in tables 4 and 5, except the values are for impedance rather than reflection coefficient.

Table 4. Calibration of Γ at 1 MHz; with adapter connected to LCR meter.

Device	"Estimated" meter readings		Known standard values		Difference	
	$R_e(\Gamma_{2u})$	$I_m(\Gamma_{2u})$	$R_e(\Gamma_2)$	$I_m(\Gamma_2)$	$R_e(\Gamma_{2u}) - R_e(\Gamma_2)$	$I_m(\Gamma_{2u}) - I_m(\Gamma_2)$
Short	-1.00046	0.00084	-1.00000	0.00000	-0.00046	0.00084
	0.00114*	0.00114*	0.00000†	0.00000†		
50 Ω	0.00130	-0.00012	0.00025	0.00087	0.00105	-0.00099
	0.00103	0.00103	0.00050	0.00040		
100 Ω	0.33363	-0.00081	0.33258	-0.00088	0.00105	0.00007
	0.00103	0.00103	0.00045	0.00045		
Open	0.99961	-0.00094	1.00000	-0.00063	-0.00039	-0.00031
	0.00111	0.00111	0.00000	0.00000		
1000 pF	0.82002	-0.57138	0.82016	-0.57214	-0.00014	0.00076
	0.00110	-0.00110	0.00004	0.00006		
1 μH	-0.96781	0.23932	-0.96789	0.23860	0.00008	0.00072
	0.00119	0.00119	0.00025	0.00027		
2.5 μH	-0.81647	0.56596	-0.81709	0.56574	0.00062	0.00022
	0.00126	0.00126	0.00061	0.00065		
5 μH	-0.44983	0.88492	-0.44740	0.88586	-0.00243	0.00093
	0.00133	0.00133	0.00103	0.00091		
10 μH	0.17369	0.97696	0.17351	0.97691	0.00018	0.00005
	0.00132	0.00132	0.00108	0.00081		
25 μH	0.79441	0.59814	0.79400	0.59853	0.00041	-0.00039
	0.00120	0.00120	0.00072	0.00075		

*These entries are $S(\Gamma_{2u})$ which are the standard deviations obtained from propagation of errors.

†These entries are estimated limits to the systematic error in the standard values.

Table 5. Calibration of Γ at 10 MHz.

Device	"Estimated" meter readings		Known standard values		Difference	
	$R_e(\Gamma_{2u})$	$I_m(\Gamma_{2u})$	$R_e(\Gamma_2)$	$I_m(\Gamma_2)$	$R_e(\Gamma_{2u}) - R_e(\Gamma_2)$	$I_m(\Gamma_{2u}) - I_m(\Gamma_2)$
Short	-1.00204	0.00338	-1.00000	0.00000	-0.00204	0.00338
	0.00354*	0.00354*	0.00000†	0.00000†		
50 Ω	0.00396	0.00066	0.00064	0.00003	0.00332	0.00063
	0.00305	0.00305	0.00049	0.00008		
100 Ω	0.33623	-0.00539	0.33307	-0.00557	0.00316	0.00018
	0.00305	0.00305	0.00045	0.00045		
Open	0.99965	-0.00771	0.99998	-0.00628	-0.00033	-0.00143
	0.00343	0.00343	0.00000	0.00001		
1000 pF	-0.82613	-0.56385	-0.82755	-0.56139	0.00142	-0.00246
	0.00363	0.00363	0.00123	0.00182		
1 μH	-0.22398	0.97246	0.22774	0.97323	-0.00376	-0.00077
	0.00359	0.00359	0.00092	0.00030		
200 pF	0.43176	-0.90048	0.43364	-0.90108	-0.00188	0.00054
	0.00359	0.00359	0.00061	0.00030		

---

*These entries are $S(\Gamma_{2u})$ which are the standard deviations obtained from propagation of errors.

†These entries are estimated limits to the systematic error in the standard values.

Table 6. Calibration of Z at 1 MHz.

Device	"Estimated" meter readings		Known standard values		Difference	
	$R_e(Z_{2u})$	$I_m(Z_{2u})$	$R_e(Z_2)$	$I_m(Z_2)$	$R_e(Z_{2u}) - R_e(Z_2)$	$I_m(Z_{2u}) - I_m(Z_2)$
Short	-0.01155	0.02090	0.00000	0.00000	-0.01155	0.02090
	0.02849*	0.02849*	0.00000†	0.00000†		
50 Ω	50.13004	-0.01198	50.02500	0.08730	0.10504	-0.09928
	0.10309	0.10309	0.05000	0.04000		
100 Ω	100.06759	-0.18219	99.83000	-0.19790	0.23759	0.01571
	0.23120	0.23120	0.10000	0.10000		
Open	3.72E+4	-9.05E+4	0.00000	-1.59E+5	3.74E+4	6.85E+4
	10.62E+5	10.62E+5	0.00000	100.00000		
1000 pF	0.15153	-159.21656	0.00000	-159.06700	0.15153	-0.14956
	0.30698	0.30698	0.00000	0.02000		
1 µH	0.07729	6.09026	0.07970	6.07200	-0.00241	0.01826
	0.03018	0.03018	0.00500	0.00600		
2.5 µH	0.18061	15.63470	0.16980	15.62000	0.01081	0.01470
	0.03477	0.03477	0.01000	0.01500		
5 µH	0.25240	30.67236	0.26180	30.76200	-0.00940	-0.08964
	0.04606	0.04606	0.02000	0.03000		
10 µH	0.46987	59.67123	0.47440	59.66000	-0.00453	0.01023
	0.08058	0.08058	0.04000	0.06000		
25 µH	1.39296	149.52158	1.41370	149.38000	-0.02074	0.14158
	0.29879	0.29879	0.11000	0.15000		

*These entries are $S(\Gamma_{2u})$ which are the standard deviations obtained from propagation of errors.

†These entries are estimated limits to the systematic error in the standard values.

Table 7. Calibration of Z at 10 MHz with adapter connected directly to LCR meter.

Device	"Calibrated" meter readings		Known standard values		Difference	
	$R_e(Z_{2u})$	$I_m(Z_{2u})$	$R_e(Z_2)$	$I_m(Z_2)$	$R_e(Z_{2u}) - R_e(Z_2)$	$I_m(Z_{2u}) - I_m(Z_2)$
Short	-0.05119	0.08415	0.00000	0.00000	-0.05119	0.08415
	0.08826	0.08826	0.00000	0.00000		
50 Ω	50.39772	0.06657	50.06400	0.00294	0.33372	0.06363
	0.30834	0.30834	0.05000	0.00080		
100 Ω	100.64417	-1.22179	99.93000	-0.12520	0.71417	0.03021
	0.69511	0.69511	0.10000	0.10000		
Open	539.75235	-12.94E+3	0.00000	-1.5915E+4	539.75235	29.70E+2
	5.78E+2	5.78E+2	0.00000	15.00000		
1000 pF	-0.00564	-15.43647	0.00000	-15.35900	-0.00564	-0.07747
	0.09924	0.09924	0.00000	0.06000		
1 μH	0.13374	62.82566	0.3070	63.05100	-0.10304	-0.22534
	0.23198	0.23198	0.00600	0.06000		
200 pF	0.11461	-79.42231	0.00000	-79.55100	0.11466	0.12869
	0.31685	0.31685	0.00000	0.06000		

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*These entries are $S(\Gamma_{2u})$ which are the standard deviations obtained from propagation of errors.

[†] These entries are estimated limits to the systematic error in the standard values.

Table 8. Least-squares estimates of calibration parameters with 1 m cable inserted.

	1 MHz		10 MHz	
	Real	Imaginary	Real	Imaginary
α	0.99965	-0.00286	0.99883	-0.02638
S_α	0.00033	0.00033	0.00089	0.00089
β	0.00055	0.00033	0.00972	0.00967
S_β	0.00034	0.00034	0.00068	0.00068
γ	0.00052	-0.00144	0.00802	-0.00779
S_γ	0.00044	0.00044	0.00849	0.00849
Residual standard deviation $\hat{\sigma}$	0.00053		0.00145	
Degrees of freedom	4		8	

7.5 Calibration Parameters with a 1 m Cable

To further demonstrate the estimation procedure, data were obtained at 1 and 10 MHz with a 1 m remote measurement harness inserted between the front panel of the meter and the adapter. Only the resulting parameter estimates are given. These are presented in table 8. At 1 MHz data were not obtained for the inductors of 1, 2.5, 5, 10, and 25 μH, therefore only five standards were used.

8. Summary

A method has been presented which permits a calibration of the LCR meter when an adapter is connected to the test port. This allows the measurement of impedance for devices with 14 mm precision coaxial connectors.

The estimation procedures for the parameters and associated uncertainties described in this report assume that the errors are random (see sect. 5.2). However, application of the method on actual data indicates that this is probably not the case. The propagation of error formula used to estimate the various standard deviations; i.e., $S(\Gamma_{2u})$ and $S(Z_{2u})$ depend on the nature of randomness in these errors. While in actuality these uncertainty estimates are reasonable bounds based on practical considerations, they do not have the same interpretation statistically, as if the errors were random.

The major contribution to the systematic error is from the uncertainty in the values of the standards. Recall in section 5.2 it was assumed this error is negligible. Examination of the error limits for the values of the standard indicate that this error is not negligible as in many cases it is the same order of magnitude as the residual standard deviation computed from eq (6-2) and given in table 3.

For example, at 1 MHz the computed residual standard deviation given in table 3 is 0.00096. In the event the errors are random this is an estimate of the "average" error in Γ_{1i} or the measured reflection coefficient. However, we see from table 4 that the uncertainties for the real and imaginary parts of the standard reflection coefficient values for the inductor are the same order of magnitude (0.00025 to 0.00108). It is beyond the scope of this paper to address how to accomodate the errors in the standards. This will be done in future work.

At this point it is not clear how the procedure, even if successful, might be utilized and made practical to the measurement community. A requirement to use the procedure would be computer capable of solving nonlinear systems of equations. Additionally, future work should be done to detrmine the minimum number of standards needed at each frequency to carry out a reliable calibration.

We wish to acknowledge the help given us by Cletus A. Hoer who suggested the theoretical approach that was used in developing this procedure and also the assistance of Stanley R. Booker of the Sandia Corporation, Albuquerque, New Mexico, for a great deal of technical support and advice during the work. Finally, we acknowledge the support of the Sandia Corporation which funded the project and thank them for it.

9. References

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Appendix A. FORTRAN Program Listing

```
      PROGRAM LCR(INPUT,OUTPUT,TAPE5,TAPE6=OUTPUT)
      DIMENSION Y(20),XM(20,1),RES(20),COEF(6),YI(20),XMI(20,6),PAR(6)
      REAL STP(6),STOPSS,STOPP,SCALE(6),DELTA
      DIMENSION PV(20),SDPV(20),SDRES(20),VCV(6,6),DIFZ(20)
      DIMENSION VCVXHAT(2,2,10),VCVZ(2,2,10),XS(20),ZS(20),DIFG(20)
      DIMENSION DEVICE(10),ZYM(20),ZHAT(20),RESZ(20),GHAT(20),
     1ZXM(20,1),INDX(10)
      COMPLEX XHAT(10),ZXHAT(10)
      DOUBLE PRECISION DSTAK(1000)
      COMMON /CSTAK/DSTAK
      EXTERNAL MODEL
      LDSTAK=1000
      IXM=20
      READ(5,105) FREQ,N
  105 FORMAT(A10,/,I2)
      DO 101 I=1,N
      IND=2*(I-1)+1
      INDX(I)=IND
      READ(5,102) ZXM(IND,1),ZXM(IND+1,1),ZYM(IND),ZYM(IND+1),DEVICE(I)
  102 FORMAT(4F10.0,A10)
  101 CONTINUE
      NN=2*N
      CALL REFL(ZYM,ZXM,Y,XM,N,NN,INDX)
      DO 10 I=1,N
      IND=(I-1)*2+1
C*****************************************************************************
C
C     GET INITIAL ESTIMATES FROM LINEAR FIT
C
C*****************************************************************************
      XMI(I,1)=XM(IND,1)
      XMI(I+N,4)=XM(IND,1)
      XMI(I,2)=1.0
      XMI(I+N,5)=1.0
      YI(I)=Y(IND)
      YI(I+N)=Y(IND+1)
      XMI(I,4)=-1.0*XM(IND+1,1)
      XMI(I+N,1)=XM(IND+1,1)
      XMI(I,5)=0.
      XMI(I+N,2)=0.
      XMI(I,3)=-1.0*(Y(IND)*XM(IND,1)-Y(IND+1)*XM(IND+1,1))
      XMI(I+N,6)=XMI(I,3)
      XMI(I,6)=Y(IND+1)*XM(IND,1)+Y(IND)*XM(IND+1,1)
      XMI(I+N,3)=-1.0*XMI(I,6)
   10 CONTINUE
      WRITE(6,2)
    2 FORMAT(1H1)
      DO 11 I=1,NN
      WRITE(6,3) (XMI(I,J),J=1,6),YI(I)
    3 FORMAT(* *,7(F13.5,3X))
   11 CONTINUE
      WRITE(6,2)
      CALL LLSS(YI,XMI,NN,20,6,RES,LDSTAK,2111,PAR,RSD,PV,SDPV,
     1SDRES,VCV,6)
      COEF(1)=PAR(1)
      COEF(2)=PAR(4)
      COEF(3)=PAR(2)
      COEF(4)=PAR(5)
      COEF(5)=PAR(3)
      COEF(6)=PAR(6)
      WRITE(6,2)
      CALL NLSS(Y,XM,NN,1,20,MODEL,COEF,6,RES,LOSTAK,-1,-1.0,-1,-1.,
     1-1.,-1.,-1.,1,11112,6,RSD,PV,SDPV,SDRES,VCV,6)
      CALL PREPRG(COEF,Y,XM,N,VCV,RSD,XHAT,VCVXHAT)
      CALL PRERRZ(N,XHAT,VCVXHAT,ZXHAT,VCVZ)
      DO 100 I=1,N
      IND=2*I-1
      XS(IND) = REAL(XHAT(I))
      XS(IND+1) = AIMAG(XHAT(I))
      ZS(IND) = REAL(ZXHAT(I))
      ZS(IND+1) = AIMAG(ZXHAT(I))
  100 CONTINUE
      DO 12 I=1,NN
      GHAT(I)=Y(I)-RES(I)
   12 CONTINUE
      CALL IMPED(GHAT,ZHAT,N,NN,INDX)
      DO 20 I=1,NN
      RESZ(I)=ZYM(I)-ZHAT(I)
      DIFG(I) = XM(I,1) - XS(I)
      DIFZ(I) = ZXM(I,1) - ZS(I)
   20 CONTINUE
      WRITE(6,5)
    5 FORMAT(1H1,27X,*REFLECTION COEFFICIENT*,//)
      WRITE(6,6)
    6 FORMAT(* *,//,* *,28X,*PREDICTED*,37X,*ESTIMATED*,7X,
     1*STANDARD*,/,* *,12X,*LCR READING*,4X,*LCR READING*,6X,
     2*RESIDUAL*,7X,*STANDARD*,7X,*STANDARD*,7X,*DEVIATION*,6X,
     3*DIFFERENCE*)
      DO 30 I=1,N
      WRITE(6,7) DEVICE(I),Y(INDX(I)),GHAT(INDX(I)),RES(INDX(I)),
     1XM(INDX(I),1),XS(INDX(I)),SQRT(VCVXHAT(1,1,I)),DIFG(INDX(I)),
     2               Y(INDX(I)+1),GHAT(INDX(I)+1),RES(INDX(I)+1),
     3XM(INDX(I)+1,1),XS(INDX(I)+1),SQRT(VCVXHAT(2,2,I)),
     4DIFG(INDX(I)+1)
    7 FORMAT(* *,/,* *,A10,7(3X,F9.5,3X),/,* *,10X,7(3X,F9.5,3X),/)
   30 CONTINUE
      WRITE(6,8)
    8 FORMAT(1H1,//* *,34X,*IMPEDANCE*,//)
      WRITE(6,6)
      DO 40 I=1,N
      WRITE(6,9) DEVICE(I),ZYM(INDX(I)),ZHAT(INDX(I)),RESZ(INDX(I)),
     1ZXM(INDX(I),1),ZS(INDX(I)),SQRT(VCVZ(1,1,I)),DIFZ(INDX(I)),
     2              ZYM(INDX(I)+1),ZHAT(INDX(I)+1),RESZ(INDX(I)+1),
     3ZXM(INDX(I)+1,1),ZS(INDX(I)+1),SQRT(VCVZ(2,2,I)),
     4DIFZ(INDX(I)+1)
    9 FORMAT(* *,/,* *,A10,7(F14.5,1X),/,* *,10X,7(F14.5,1X),/)
   40 CONTINUE
      STOP
      END
      SUBROUTINE REFL(Y,XM,GY,GX,N,NN,IND)
      DIMENSION Y(NN),XM(NN,1),GY(NN),GX(NN,1),IND(N)
      COMPLEX CZX(10),CZY(10),GAMMAX,GAMMAY,ZO
      DO 10 I=1,N
      IND(I)=2*(I-1)+1
      CZX(I)=CMPLX(XM(IND(I),1),XM(IND(I)+1,1))
      CZY(I)=CMPLX(Y(IND(I)),Y(IND(I)+1))
   10 CONTINUE
      ZO=CMPLX(50.,0.)
      DO 20 I=1,N
      GAMMAX=(CZX(I)-ZO)/(CZX(I)+ZO)
      GAMMAY=(CZY(I)-ZO)/(CZY(I)+ZO)
      GX(IND(I),1)=REAL(GAMMAX)
      GX(IND(I)+1,1)=AIMAG(GAMMAX)
      GY(IND(I))=REAL(GAMMAY)
      GY(IND(I)+1)=AIMAG(GAMMAY)
   20 CONTINUE
      RETURN
      END
      SUBROUTINE IMPED(Y,Z,N,NN,IND)
       DIMENSION Y(NN),Z(NN),IND(N)
       COMPLEX ZY,CY(10)
       DO 10 I=1,N
      IND(I)=2*(I-1)+1
       CY(I)=CMPLX(Y(IND(I)),Y(IND(I)+1))
   10 CONTINUE
      ZO=CMPLX(50.,0.)
      DO 20 I=1,N
      ZY=ZO*(1.+CY(I))/(1.-CY(I))
      Z(IND(I))=REAL(ZY)
      Z(IND(I)+1)=AIMAG(ZY)
   20 CONTINUE
      RETURN
      END
      SUBROUTINE MODEL(COEF,NCOEF,XM,NN,M,IXM,PV)
      REAL COEF(6),XM(NN,1),PV(NN)
      DO 5 I=1,NN,2
      AXRR = COEF(1)*XM(I,1)
      AXII = COEF(2)*XM(I+1,1)
      AXIR = COEF(2)*XM(I,1)
      AXRI = COEF(1)*XM(I+1,1)
      CXRR = COEF(5)*XM(I,1)
      CXII = COEF(6)*XM(I+1,1)
      CXRI = COEF(5)*XM(I+1,1)
      CXIR = COEF(6)*XM(I,1)
      BR=COEF(3)
      BI=COEF(4)
      CREAL=CXRR-CXII+1.0
      CIMAG=CXIR+CXRI
      PV(I)=((AXRR-AXII+BR)*CREAL+(AXIR+AXRI+BI)*CIMAG)
     1/(CREAL**2 + CIMAG**2)
      PV(I+1)=((AXIR+AXRI+BI)*CREAL-(AXRR-AXII+BR)*CIMAG)
     1/(CREAL**2 + CIMAG**2)
    5 CONTINUE
      RETURN
      END
```

```
SUBROUTINE PREPRG      74/175  OPT=1  PMDMP                 FTN 4.8+552          84/05/01. 11.17.43        PAGE     1

                        SUBROUTINE PREPRG(BPAR,Y,X,N,VCV,RSD,XHAT,VCVXHAT)
                        DIMENSION BPAR(6),Y(20),X(20),VCV(6,6)
                        DIMENSION RJAK(2,8),QF1(2,8),SIG(8,8),SIGX(2,2),VCVXHAT(2,2,10)
                        COMPLEX A,B,C,YC(10),ANUM,DEN,XC(10),XHAT(10)
                        COMPLEX DXDA,DXDB,DXDC,DXDY
                        A = CMPLX(BPAR(1),BPAR(2))
                        B = CMPLX(BPAR(3),BPAR(4))
                        C = CMPLX(BPAR(5),BPAR(6))
                        DO 10 I=1,N
                        IND = 2*(I-1)+1
                        YC(I) = CMPLX(Y(IND),Y(IND+1))
                     10 CONTINUE
                        DO 20 I=1,N
                        ANUM = B-YC(I)
                        DEN = YC(I)*C-A
                        XHAT(I) = ANUM / DEN
                        DXDA = ANUM / (DEN**2)
                        DXDB = 1. / DEN
                        DXDC = (-YC(I)*(B-YC(I)))/(DEN**2)
                        DXDY = (A - B*C)/(DEN**2)
                        DO 30 J=1,8
                        DO 40 K=1,8
                        SIG(J,K) = 0.0
                     40 CONTINUE
                     30 CONTINUE
                        DO 50 J=1,6
                        DO 60 K=1,6
                        SIG(J,K) = VCV(J,K)
                     60 CONTINUE
                     50 CONTINUE
                        DO 70 J=1,2
                        SIG(J+6,J+6) = RSD**2
                     70 CONTINUE
                        RJAK(1,1) = REAL(DXDA)
                        RJAK(1,2) = AIMAG(DXDA)
                        RJAK(2,1) = -AIMAG(DXDA)
                        RJAK(2,2) = REAL(DXDA)
                        RJAK(1,3) = REAL(DXDB)
                        RJAK(1,4) = AIMAG(DXDB)
                        RJAK(2,3) = -AIMAG(DXDB)
                        RJAK(2,4) = REAL(DXDB)
                        RJAK(1,5) = REAL(DXDC)
                        RJAK(1,6) = AIMAG(DXDC)
                        RJAK(2,5) = -AIMAG(DXDC)
                        RJAK(2,6) = REAL(DXDC)
                        RJAK(1,7) = REAL(DXDY)
                        RJAK(1,8) = AIMAG(DXDY)
                        RJAK(2,7) = -AIMAG(DXDY)
                        RJAK(2,8) = REAL(DXDY)
                        CALL VMULFF(RJAK,SIG,2,8,8,2,8,QF1,2,IER)
                        CALL VMULFP(QF1,RJAK,2,8,2,2,2,SIGX,2,IER)
                        DO 80 J=1,8
                        DO 90 K=1,8
                        VCVXHAT(J,K,I) = SIGX(J,K)
                     90 CONTINUE
                     80 CONTINUE
                     20 CONTINUE
                        RETURN
                        END
```

```
SUBROUTINE PRERR7      74/175  OPT=1  PMDMP                 FTN 4.8+552        84/05/01. 11.17.43        PAGE      1

      SUBROUTINE PRERR7(N,X,VCV,Z,VCVZ)
      DIMENSION VCV(2,2,10),VCVZ(2,2,10),QF1(2,2),SIG(2,2)
      DIMENSION SIGZ(2,2),RJAK(2,2)
      COMPLEX X(10),Z(10),DZDX,ANUM,DEN
      DO 10 I=1,N
      ANUM = 50. * (1. + X(I))
      DEN = 1. - X(I)
      Z(I) = ANUM / DEN
      DZDX = 100. / ( DEN**2)
      DO 30 J=1,2
      DO 40 K=1,2
      SIG(J,K) = VCV(J,K,I)
   40 CONTINUE
   30 CONTINUE
      RJAK(1,1) = REAL(DZDX)
      RJAK(1,2) = AIMAG(DZDX)
      RJAK(2,1) =-AIMAG(DZDX)
      RJAK(2,2) = REAL(DZDX)

      CALL VMULFF(RJAK,SIG,2,2,2,2,2,QF1,2,IER)
      CALL VMULFP(QF1,RJAK,2,2,2,2,2,SIGZ,2,IER)
      DO 50 J=1,2
      DO 60 K=1,2
      VCVZ(J,K,I) = SIGZ(J,K)
   60 CONTINUE
   50 CONTINUE
   10 CONTINUE
      RETURN
      END
```

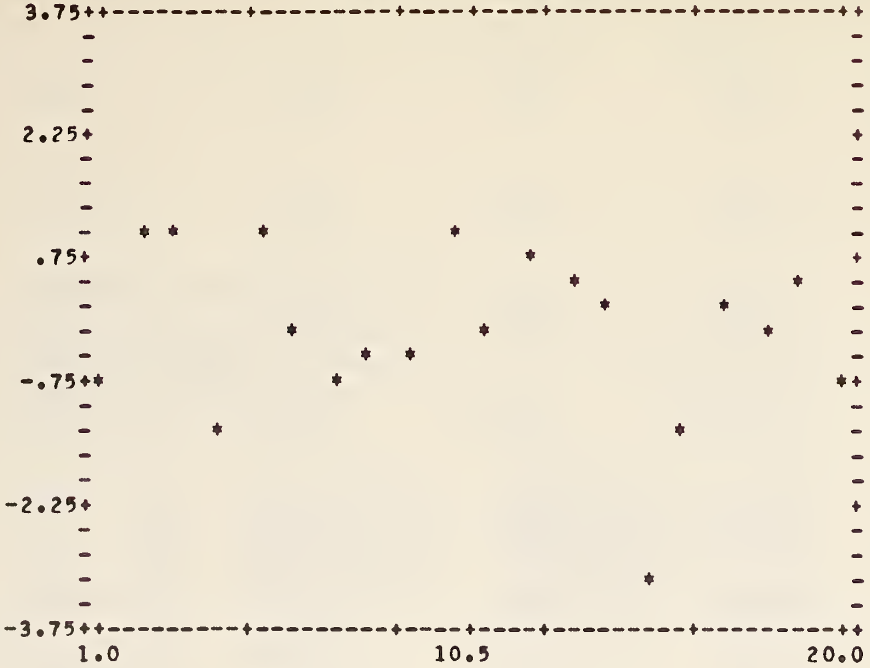
Appendix B. Printout from Program for Data at 1 MHz

```
******************************************************************************
*  NONLINEAR LEAST SQUARES ESTIMATION WITH NUMERICALLY APPROXIMATED DERIVATIVES  *
******************************************************************************                STARPAC 1.05S - MARCH 6, 1984


SUMMARY OF INITIAL CONDITIONS
-----------------------------


                                                          STEP SIZE FOR    OBSERVATIONS FAILING STEP SIZE SELECTION CRITERIA
                                                         APPROXIMATING                       *
      PARAMETER STARTING VALUE       SCALE                 DERIVATIVE        COUNT     NOTES      ROW NUMBER
INDEX  FIXED        (PAR)           (SCALE)                 (STP)                       F C

  1      NO     .99983286          DEFAULT          .24177224E-04          0
  2      NO    -.21787324E-02      DEFAULT          .24177224E-04          0
  3      NO    -.64927592E-03      DEFAULT          .24177224E-04          1                   10
  4      NO     .66179193E-03      DEFAULT          .24177224E-04          2                    1    9
  5      NO    -.12051309E-02      DEFAULT          .24177224E-04          1                    4
  6      NO    -.11060094E-02      DEFAULT          .24177224E-04          2                    3    7

*  NOTES.  A PLUS (+) IN THE COLUMNS HEADED F OR C HAS THE FOLLOWING MEANING.

   F - NUMBER OF OBSERVATIONS FAILING STEP SIZE SELECTION CRITERIA EXCEEDS
       NUMBER OF EXEMPTIONS ALLOWED.

   C - HIGH CURVATURE IN THE MODEL IS SUSPECTED AS THE CAUSE OF
       ALL FAILURES NOTED.

NUMBER OF RELIABLE DIGITS IN MODEL RESULTS                          (NETA)   13

PROPORTION OF OBSERVATIONS EXEMPTED FROM SELECTION CRITERIA        (EXMPT)  .1000

NUMBER OF OBSERVATIONS EXEMPTED FROM SELECTION CRITERIA                      2

NUMBER OF OBSERVATIONS                                                 (N)   20

NUMBER OF INDEPENDENT VARIABLES                                        (M)    1

MAXIMUM NUMBER OF ITERATIONS ALLOWED                                 (MIT)   21

MAXIMUM NUMBER OF MODEL SUBROUTINE CALLS ALLOWED                             42

CONVERGENCE CRITERION FOR TEST BASED ON THE

     FORECASTED RELATIVE CHANGE IN RESIDUAL SUM OF SQUARES        (STOPSS)  .3696E-09
     MAXIMUM SCALED RELATIVE CHANGE IN THE PARAMETERS              (STOPP)  .8425E-07


MAXIMUM CHANGE ALLOWED IN THE PARAMETERS AT THE FIRST ITERATION    (DELTA)  100.0

RESIDUAL SUM OF SQUARES FOR INPUT PARAMETER VALUES                          .1298E-04

RESIDUAL STANDARD DEVIATION FOR INPUT PARAMETER VALUES               (RSD)  .9527E-03
```

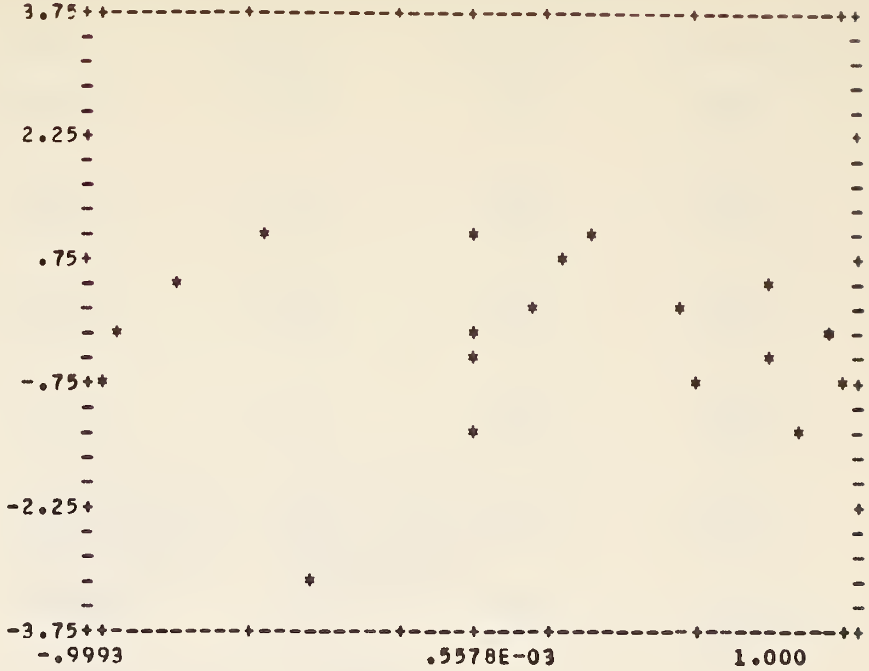
```
RESULTS FROM LEAST SQUARES FIT
------------------------------

                                                     DEPENDENT        PREDICTED       STD DEV OF                         STD
 ROW             PREDICTOR VALUES                    VARIABLE           VALUE         PRED VALUE        RESIDUAL         RES

   1              -1.0000000                        -.99973022       -.99927342      .60367921E-03  -.45679892E-03     -.61
   2               0.                                .47767383E-02    .39404679E-02  .60368000E-03   .83627041E-03     1.12
   3                .25069907E-03                    .64985881E-03   -.39579124E-03  .36064664E-03   .10456500E-02     1.17
   4                .87256300E-03                    .53929853E-03    .15334217E-02  .36064740E-03  -.99412316E-03    -1.11
   5                .33257808                        .33306296        .33200570      .35864060E-03   .10572545E-02     1.18
   6               -.88155107E-03                   -.75172143E-03   -.82279156E-03  .35864105E-03   .71070126E-04      .08
   7                .99999980                        .99999955       1.0003890       .55500617E-03  -.38943048E-03     -.50
   8               -.62893076E-03                   -.13572203E-02   -.10407335E-02  .55500581E-03  -.31648677E-03     -.40
   9                .82015883                        .81944269        .81958042      .70372369E-03  -.13772847E-03     -.21
  10               -.57213590                       -.57316113       -.57391594      .70372139E-03   .75480679E-03     1.15
  11               -.96798761                       -.96620135       -.96628358      .53010822E-03   .82323014E-04      .10
  12                .23859994                        .24245828        .24174132      .53010912E-03   .71695563E-03      .89
  13               -.81709247                       -.81431060       -.81493357      .47043147E-03   .62296851E-03      .74
  14                .56573844                        .56756677        .56735335      .47043062E-03   .21341790E-03      .25
  15               -.44740355                       -.44829270       -.44587659      .49493393E-03  -.24161078E-02    -2.93
  16                .88586218                        .88481812        .88574987      .49493610E-03  -.93175282E-03    -1.13
  17                .17351279                        .17365329        .17347613      .56055822E-03   .17715874E-03      .23
  18                .97691212                        .97646364        .97641824      .56055606E-03   .45399380E-04      .06
  19                .79399708                        .79420993        .79379523      .54083506E-03   .41470102E-03      .52
  20                .59853145                        .59841302        .59880858      .54083670E-03  -.39555772E-03     -.50
```

NONLINEAR LEAST SQUARES ESTIMATION WITH NUMERICALLY APPROXIMATED DERIVATIVES, CONTINUED STARPAC 1.05S - MARCH 6, 1984

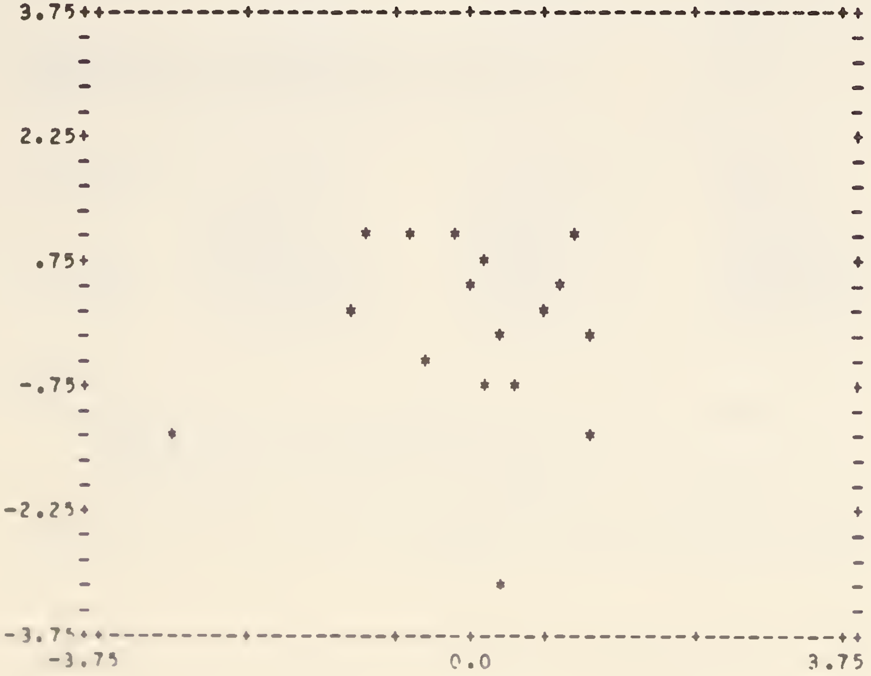
STD RES VS ROW NUMBER



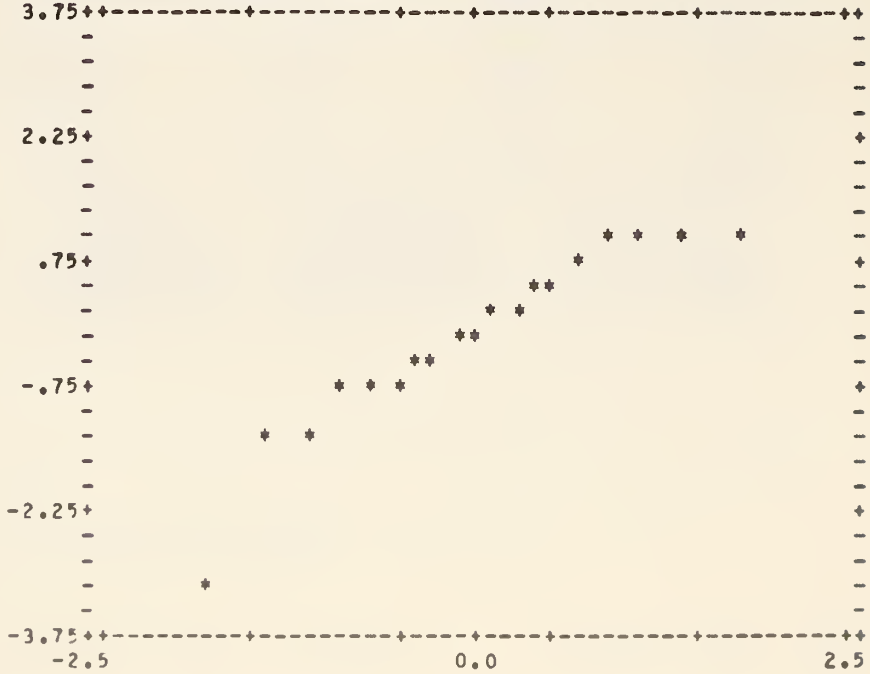
STD RES VS PREDICTED VALUES



STD RES (I) VS STD RES (I-1)



NORMAL PROBABILITY PLOT OF STD RES



```
NONLINEAR LEAST SQUARES ESTIMATION WITH NUMERICALLY APPROXIMATED DERIVATIVES, CONTINUED          STARPAC 1.05S - MARCH 6, 1984


VARIANCE-COVARIANCE AND CORRELATION MATRICES OF THE ESTIMATED (UNFIXED) PARAMETERS
----------------------------------------------------------------------------------

   - APPROXIMATION BASED ON ASSUMPTION THAT RESIDUALS ARE SMALL
   - COVARIANCES ARE ABOVE THE DIAGONAL
   - VARIANCES ARE ON THE DIAGONAL
   - CORRELATION COEFFICIENTS ARE BELOW THE DIAGONAL

COLUMN         1                2                3                4                5                6

      1     .1607426E-06     .1397059E-12    -.1158859E-07    -.6232540E-07     .3220987E-08    -.7599177E-07
      2     .8691315E-06     .1607413E-06     .6232449E-07    -.1158842E-07     .7598988E-07     .3220294E-08
      3    -.8011101E-01     .4308460         .1301804E-05     .1726646E-12     .6180603E-07     .3119581E-07
      4    -.4308498        -.8011001E-01     .1326345E-05     .1301810E-06    -.3119581E-07     .6180738E-07
      5     .1952035E-01     .4605280         .4162191        -.2100809         .1693838E-06    -.7721998E-12
      6    -.4605346         .1951609E-01     .2100799         .4162245        -.4558845E-05     .1693861E-06


ESTIMATES FROM LEAST SQUARES FIT
--------------------------------


                                                                        APPROXIMATE
                                                                95 PERCENT CONFIDENCE LIMITS
INDEX  FIXED    PARAMETER           SD OF PAR        RATIO            LOWER             UPPER

  1      NO      .99983257          .40092712E-03    2494.          .99897252         1.0006926
  2      NO    -.21781717E-02       .40092558E-03   -5.433        -.30382203E-02     -.13181231E-02
  3      NO    -.64834716E-03       .36080526E-03   -1.797        -.14223313E-02      .12563700E-03
  4      NO     .66155239E-03       .36080602E-03    1.834        -.11243340E-03      .14355382E-02
  5      NO    -.12040108E-02       .41156265E-03   -2.925        -.20868776E-02     -.32114407E-03
  6      NO    -.11062920E-02       .41156543E-03   -2.688        -.19891648E-02     -.22341931E-03

RESIDUAL SUM OF SQUARES                         .1297513E-04

RESIDUAL STANDARD DEVIATION                     .9627019E-03
BASED ON DEGREES OF FREEDOM          20 -    6 =    14

APPROXIMATE CONDITION NUMBER                    1.351489
```

```
                     REFLECTION COEFFICIENT

                              PREDICTED                                    ESTIMATED     STANDARD
              LCP READING    LCP READING    RESIDUAL      STANDARD      STANDARD     DEVIATION     DIFFERENCE

SHORT           -.99973        -.99927       -.00046      -1.00000      -1.00046       .00114        .00046
                 .00478         .00394        .00084       0.00000        .00084       .00114       -.00084

50 OHM           .00065        -.00040        .00105        .00025        .00130       .00103       -.00105
                 .00054         .00153       -.00099        .00087       -.00012       .00103        .00099

100 OHM          .33306         .33201        .00106        .33258        .33363       .00103       -.00106
                -.00075        -.00082        .00007       -.00088       -.00081       .00103       -.00007

OPEN            1.00000        1.00039       -.00039       1.00000        .99961       .00111        .00039
                -.00136        -.00104       -.00032       -.00063       -.00094       .00111        .00032

1000 PF          .81944         .81958       -.00014        .82016        .82002       .00110        .00014
                -.57316        -.57392        .00075       -.57214       -.57138       .00110       -.00075

1 MICRO H       -.96620        -.96628        .00008       -.96789       -.96781       .00119       -.00008
                 .24246         .24174        .00072        .23860        .23932       .00119       -.00072

2.5 MICROH      -.81431        -.81493        .00062       -.81709       -.81647       .00126       -.00062
                 .56757         .56735        .00021        .56574        .55595       .00126       -.00022

5 MICRO H       -.44829        -.44588       -.00242       -.44740       -.44983       .00133        .00242
                 .88482         .88575       -.00093        .88586        .88492       .00133        .00094

10 MICRO H       .17365         .17348        .00018        .17351        .17369       .00132       -.00018
                 .97646         .97642        .00005        .97691        .97696       .00132       -.00005

25 MICRO H       .79421         .79380        .00041        .79400        .79441       .00120       -.00041
                 .59841         .59881       -.00040        .59853        .59814       .00120        .00040
```

```
                          IMPEDANCE


                             PREDICTED                                ESTIMATED       STANDARD
               LCR READING   LCR READING      RESIDUAL      STANDARD       STANDARD      DEVIATION     DIFFERENCE

SHORT               .00646        .01798       -.01152       0.00000        -.01155         .02849         .01155
                    .11945        .09858        .02087       0.00000         .02090         .02849        -.02090


50 OHM            50.06500      49.96020        .10480      50.02500       50.13004         .10309        -.10504
                    .05400        .15322       -.09922        .08730        -.01198         .10309         .09928


100 OHM           99.93900      99.70165        .23735      99.83000      100.06759         .23120        -.23759
                   -.16900       -.18439        .01539       -.19790        -.18219         .23120        -.01571


OPEN              -25.72000  -31561.05570   31535.33570       0.00000    37202.87638   106217.48951   -37202.87638
               -73680.00000  -84308.54609   10628.54609 -159000.00000   -90523.73256   106217.44630   -68476.26744


1000 PF            0.00000       -.15080        .15080       0.00000         .15153         .30698        -.15153
                -158.72000    -158.57066       -.14934    -159.06700     -159.21556         .30698         .14956


1 MICRO H           .09770        .10010       -.00240        .07970         .07729         .03018         .00241
                   6.17770       6.15947        .01823       6.07200        6.09026         .03018        -.01826


2.5 MICROH          .20430        .19350        .01080        .16980         .18061         .03477        -.01081
                  15.70530      15.69063        .01467      15.62000       15.63470         .03477        -.01470


5 MICRO H           .28000        .28940       -.00940        .26180         .25240         .04606         .00940
                  30.71800      30.80749       -.08949      30.76200       30.67236         .04606         .08964


10 MICRO H          .50000        .50453       -.00453        .47440         .46987         .08058         .00453
                  59.67400      59.65378        .01022      59.66100       59.67123         .08058        -.01023


25 MICRO H         1.39000       1.41083       -.02083       1.41370        1.39296         .29879         .02074
                 149.43600     149.29452        .14148     149.38000      149.52158         .29879        -.14158
```

Appendix C. Printout from Program for Data at 10 MHz

```
******************************************************************************                    STARPAC 1.05S - MARCH 6, 1984
*  NONLINEAR LEAST SQUARES ESTIMATION WITH NUMERICALLY APPROXIMATED DERIVATIVES  *
******************************************************************************

SUMMARY OF INITIAL CONDITIONS
-----------------------------


                                                  STEP SIZE FOR    OBSERVATIONS FAILING STEP SIZE SELECTION CRITERIA
                                                  APPROXIMATING               *
      PARAMETER STARTING VALUE       SCALE         DERIVATIVE        COUNT      NOTES     ROW NUMBER
INDEX  FIXED        (PAR)           (SCALE)          (STP)                       F C

  1      NO     .99823552           DEFAULT       .30331054E-04         0
  2      NO    -.24139722E-01       DEFAULT       .30331054E-04         0
  3      NO    -.51155919E-02       DEFAULT       .30331054E-04         0
  4      NO     .85197609E-02       DEFAULT       .30331054E-04         0
  5      NO    -.71569793E-02       DEFAULT       .30331054E-04         0
  6      NO    -.97591431E-02       DEFAULT       .30331054E-04         0

*  NOTES.  A PLUS (+) IN THE COLUMNS HEADED F OR C HAS THE FOLLOWING MEANING.

   F - NUMBER OF OBSERVATIONS FAILING STEP SIZE SELECTION CRITERIA EXCEEDS
       NUMBER OF EXEMPTIONS ALLOWED.

   C - HIGH CURVATURE IN THE MODEL IS SUSPECTED AS THE CAUSE OF
       ALL FAILURES NOTED.

NUMBER OF RELIABLE DIGITS IN MODEL RESULTS                          (NETA)    13

PROPORTION OF OBSERVATIONS EXEMPTED FROM SELECTION CRITERIA        (EXMPT)   .1000

NUMBER OF OBSERVATIONS EXEMPTED FROM SELECTION CRITERIA                        1

NUMBER OF OBSERVATIONS                                                 (N)    14

NUMBER OF INDEPENDENT VARIABLES                                        (M)     1

MAXIMUM NUMBER OF ITERATIONS ALLOWED                                 (MIT)    21

MAXIMUM NUMBER OF MODEL SUBROUTINE CALLS ALLOWED                              42

CONVERGENCE CRITERION FOR TEST BASED ON THE

     FORECASTED RELATIVE CHANGE IN RESIDUAL SUM OF SQUARES        (STOPSS)   .3696E-09
     MAXIMUM SCALED RELATIVE CHANGE IN THE PARAMETERS              (STOPP)   .8425E-07

MAXIMUM CHANGE ALLOWED IN THE PARAMETERS AT THE FIRST ITERATION    (DELTA)   100.0

 ESIDUAL SUM OF SQUARES FOR INPUT PARAMETER VALUES                          .6492E-04

 ESIDUAL STANDARD DEVIATION FOR INPUT PARAMETER VALUES               (RSD)   .2849E-02
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NONLINEAR LEAST SQUARES ESTIMATION WITH NUMERICALLY APPROXIMATED DERIVATIVES, CONTINUED          STARPAC 1.05S - MARCH 6, 1984

RESULTS FROM LEAST SQUARES FIT
------------------------------

                                                 DEPENDENT       PREDICTED      STD DEV OF                        STD
 ROW               PREDICTOR VALUES              VARIABLE          VALUE        PRED VALUE       RESIDUAL         RES
  .
   1                 -1.0000000                -.99767008      -.99780466      .20111449E-02  -.18654146E-02     -.92
   2                 0.                         .45465477E-01   .42061669E-01  .20111434E-02   .34038080E-02     1.69
   3                  .63959152E-03            -.11391122E-02  -.44704050E-02  .11021767E-02   .33312928E-02     1.27
   4                  .29362404E-04             .90813997E-02   .85315732E-02  .11021781E-02   .54982648E-03      .21
   5                  .33306858                 .33122055       .32804819      .11504787E-02   .31723681E-02     1.22
   6                 -.55692532E-02            -.39159476E-02  -.40460838E-02  .11504790E-02   .13013620E-03      .05
   7                  .99998026                 .99994699      1.0002895       .20091553E-02  -.34251833E-03     -.17
   8                 -.62833184E-02            -.13749280E-01  -.12306616E-01  .20091432E-02  -.14426641E-02     -.71
   9                 -.82755292                -.84936330      -.85066158      .19418477E-02   .12983821E-02      .62
  10                 -.56138771                -.52389592      -.52138041      .19418407E-02  -.25155072E-02    -1.21
  11                  .22774248                 .23130475       .23499662      .23289248E-02  -.36918635E-02    -2.25
  12                  .97323461                 .96835405       .96908845      .23289090E-02  -.73439952E-03     -.45
  13                  .43364350                 .40691912       .40882137      .21282573E-02  -.19022466E-02    -1.00
  14                 -.90108452                -.91259194      -.91320074      .21282429E-02   .60880009E-03      .32
```
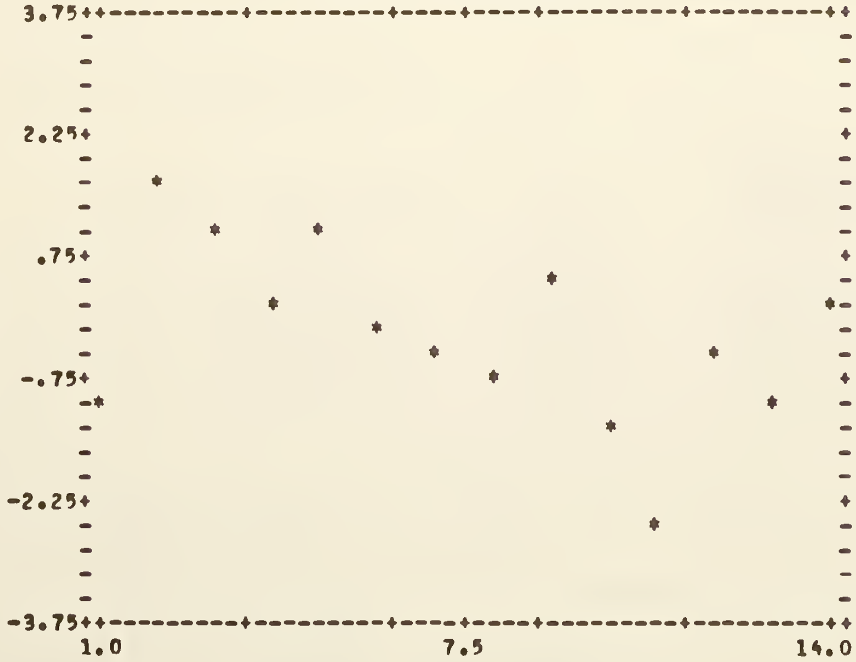
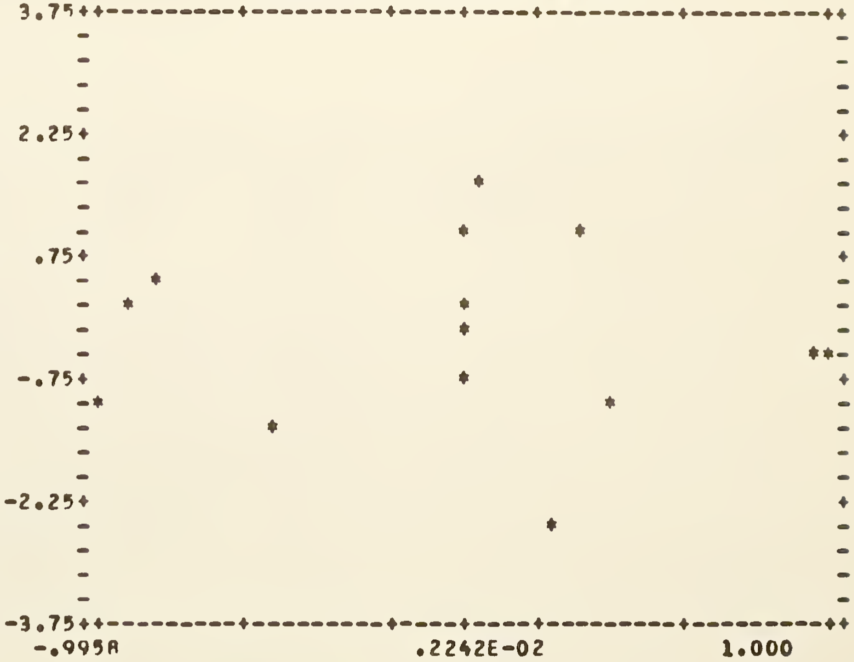
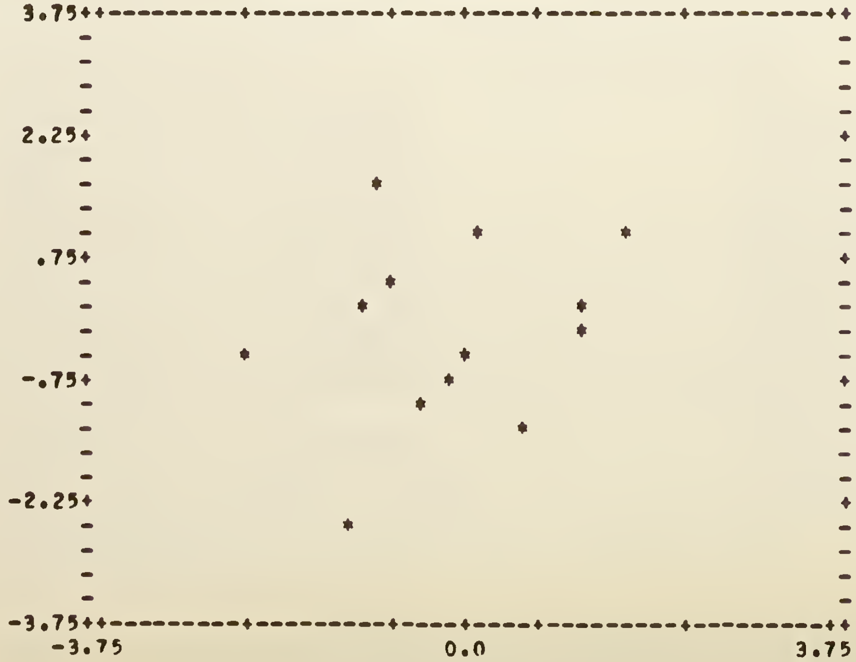
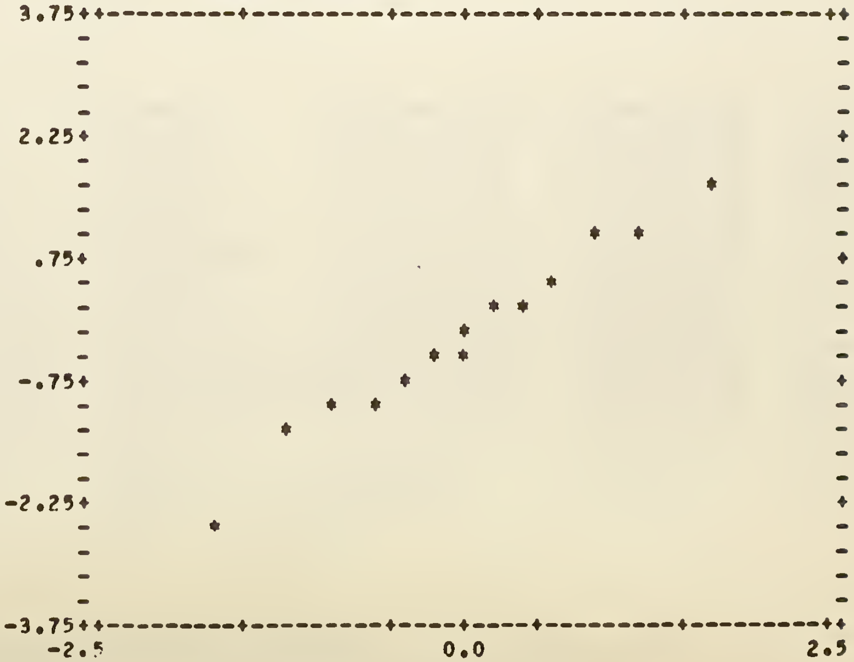

NONLINEAR LEAST SQUARES ESTIMATION WITH NUMERICALLY APPROXIMATED DERIVATIVES, CONTINUED   STARPAC 1.05S - MARCH 6, 1984


STD RES VS ROW NUMBER


STD RES VS PREDICTED VALUES


STD RES (I) VS STD RES (I-1)


NORMAL PROBABILITY PLOT OF STD RES

```
VARIANCE-COVARIANCE AND CORRELATION MATRICES OF THE ESTIMATED (UNFIXED) PARAMETERS
----------------------------------------------------------------------------------

   - APPROXIMATION BASED ON ASSUMPTION THAT RESIDUALS ARE SMALL
   - COVARIANCES ARE ABOVE THE DIAGONAL
   - VARIANCES ARE ON THE DIAGONAL
   - CORRELATION COEFFICIENTS ARE BELOW THE DIAGONAL

COLUMN         1                2                3                4                5                6

     1    .1622216E-05    -.7512985E-12    -.6685257E-07     .1479252E-06    -.4038790E-07     .1895180E-06
     2   -.4631309E-06     .1622217E-05    -.1479238E-06    -.6685402E-07    -.1895193E-06    -.4039527E-07
     3   -.4762125E-01    -.1053709         .1214859E-05    -.5537708E-12     .2535655E-06    -.1248654E-06
     4    .1053718        -.4762222E-01    -.4558308E-06     .1214862E-05     .1248670E-06     .2536751E-06
     5   -.2429997E-01    -.1140270         .1763629         .8681469E-01     .1702873E-05     .3730892E-11
     6    .1140264        -.2430442E-01    -.8681374E-01     .1763695         .2190941E-05     .1702871E-05


ESTIMATES FROM LEAST SQUARES FIT
--------------------------------


                                                                     APPROXIMATE
                                                             95 PERCENT CONFIDENCE LIMITS
INDEX  FIXED   PARAMETER        SD OF PAR       RATIO             LOWER          UPPER

  1      NO    .99823133       .12736625E-02   783.7          .99528984       1.0011728
  2      NO   -.24153616E-01   .12736627E-02  -18.96        -.27095104E-01   -.21212127E-01
  3      NO   -.51095004E-02   .11022064E-02  -4.636        -.76550154E-02   -.25639855E-02
  4      NO    .85177033E-02   .11022078E-02   7.728         .59721852E-02    .11063221E-01
  5      NO   -.71568377E-02   .13049417E-02  -5.484        -.10170564E-01   -.41431112E-02
  6      NO   -.97322083E-02   .13049409E-02  -7.458        -.12745933E-01   -.67184838E-02

RESIDUAL SUM OF SQUARES                        .6491691E-04

RESIDUAL STANDARD DEVIATION                    .2848616E-02
BASED ON DEGREES OF FREEDOM          14 -     6 =      8

APPROXIMATE CONDITION NUMBER                   1.213106
```

REFLECTION COEFFICIENT

	LCR READING	PREDICTED LCR READING	RESIDUAL	STANDARD	ESTIMATED STANDARD	STANDARD DEVIATION	DIFFERENCE
SHORT	-.99767	-.99580	-.00187	-1.00000	-1.00234	.00354	.00204
	.04547	.04206	.00340	0.00000	.00337	.00354	-.00337
50 OHM	-.00114	-.00447	.00333	.00064	.00396	.00306	-.00332
	.00908	.00853	.00055	.00003	.00065	.00306	-.00063
100 OHM	.33122	.32805	.00317	.33307	.33623	.00306	-.00316
	-.00392	-.00405	.00013	-.00557	-.00538	.00306	-.00019
OPEN	.99995	1.00029	-.00034	.99998	.99965	.00344	.00033
	-.01375	-.01231	-.00144	-.00628	-.00771	.00344	.00143
1000 PF	-.84936	-.85066	.00130	-.82755	-.82613	.00363	-.00142
	-.52390	-.52138	-.00252	-.56139	-.56384	.00363	.00246
1 MICRO H	.23130	.23500	-.00369	.22774	.22399	.00359	.00375
	.96835	.95909	-.00073	.97323	.97245	.00359	.00077
200 PF	.40692	.40882	-.00190	.43364	.43177	.00359	.00188
	-.91259	-.91320	.00061	-.90108	-.90054	.00359	-.00054

```
                                IMPEDANCE


                               PREDICTED                                      ESTIMATED        STANDARD
                LCR READING    LCR READING      RESIDUAL       STANDARD       STANDARD         DEVIATION        DIFFERENCE

SHORT                .03240         .08286       -.05046        0.00000         -.05119          .08826           .05119
                    1.13870        1.05550        .08320        0.00000          .08415          .08826          -.08415


50 OHM             49.87800       49.54777        .33023       50.06400        50.39772          .30834          -.33372
                     .90600         .84552        .06048         .00294          .06657          .30834          -.06363


100 OHM            99.52100       98.81480        .70620       99.93000       100.64417          .69511          -.71417
                    -.87550        -.89607        .02057       -1.25200        -1.22179          .69511          -.03021


OPEN              -21.96000     -241.04952     219.08952        0.00000       539.75235       5778.92551       -539.75235
               -7273.00000    -8121.21622     848.21622   -15915.00000    -12944.65957       5778.92379      -2970.34043


1000 PF              .05569         .06137       -.00568        0.00000         -.00554          .09924           .00564
                  -14.18000      -14.10361       -.07639      -15.35900       -15.43647          .09924           .07747


1 MICRO H            .28747         .18513        .10234         .03070          .13374          .23198          -.10304
                   63.34900       63.57336       -.22436       63.05100        62.82565          .23198           .22534


200 PF               .06723        -.04523        .11246        0.00000          .11466          .31685          -.11466
                  -77.04000      -77.16574        .12574      -79.55100       -79.42231          .31685          -.12869
```

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4. TITLE AND SUBTITLE

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5. AUTHOR(S)

R. M. Judish and R. N. Jones

6. PERFORMING ORGANIZATION *(If joint or other than NBS, see instructions)*

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Sandia National Laboratories
Albuquerque, New Mexico

10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)*

Since the introduction of automated, four-terminal-pair type digital impedance meters, there has been a continuing interest in the development of calibration techniques which would satisfactorily verify the accuracy capabilities of these instruments. Various attempts have been made and all have helped to provide a certain degree of confidence in instrument performance, but until now, a generalized approach with a good mathematical and statistical background has been lacking. This paper describes a calibration procedure having such a background and illustrates its use. The calibration is accomplished through the use of impedance standards which relate instrument readings to the values of the standards through a known functional relationship. The calibration procedure described estimates the parameters associated with the functional relationship and requires the use of a computer. Calibration is accomplished at the reference plane of the impedance standards and any adapter required to connect the standards to the instrument is assumed to be an integral part of the impedance meter.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

calibration; digital impedance meter; impedance; least-squares; measurement; reflection coefficient; uncertainty

13. AVAILABILITY

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